Vanderbilt Law Review

Volume 62
Issue 7 *En Banc*

Article 1

3-2009

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VANDERBILT LAW REVIEW
EN BANC

VOLUME 62 MARCH 2009

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I. INTRODUCTION

This is a Response to Professor Paul Edelman's piece, *Getting the Math Right: Why California Has Too Many Seats in the House of*

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Representatives.[1] Professor Edelman argues that in the seminal apportionment case, *U.S. Department of Commerce v. Montana*,[2] the Court relied on erroneous calculations and incorrectly decided the case. Professor Edelman's assertions are on-point insofar as the Court did miscalculate certain figures, but this Response argues that the Court should have calculated the figures in a different manner than that suggested by Professor Edelman. Calculating the figures as I suggest results in a proper distinction between relative *difference* and relative *deviation*.

This Response will introduce the apportionment problem and provide an introduction to apportionment. This Response will then demonstrate that relative *difference* can, and should, have a different, independent meaning from relative *deviation*. After accepting that there is a distinction between relative difference and deviation, this Response shows how absolute difference and relative difference are distinct and demonstrates how the Court should have calculated relative difference.

II. APPORTIONMENT IN THE AMERICAN CONTEXT

The Constitution requires representative apportionment among the several states "according to their respective Numbers."[3] Although this sounds simple in theory, the application of representative apportionment historically has been vexatious.[4] The Constitution requires representative apportionment, but does not provide a method by which to apportion.[5] This is one area where the Constitution demands the impossible: it is impossible to apportion perfectly. There are three crucial apportionment constraints: (1) congressional districts cannot cross state boundaries; (2) representatives can only be apportioned in integers; and (3) currently, the number of representatives is constrained to 435.[6] The "integer" constraint is particularly problematic and will be discussed below.

In order to understand why the "integer" constraint is so troubling, a brief history of "quota" or "fair share" is essential. Under

1. Paul H. Edelman, *Getting the Math Right: Why California Has Too Many Seats in the House of Representatives*, 59 VAND. L. REV. 297 (2006).
2. 503 U.S. 442 (1992).
3. U.S. CONST. art. I, § 2, cl. 3.
4. Paul H. Edelman & Suzanna Sherry, *Pick a Number, Any Number: State Representation in Congress After the 2000 Census*, 90 CAL. L. REV. 211, 211–12 (2002).
5. Efton Park, *The Mathematics of Apportionment*, 7 U. CHI. L. SCH. ROUNDTABLE 227 (2000).
6. *Montana*, 503 U.S. at 447–48.

the current constraint of 435 representatives, each state has claim to a particular "fair share" of representatives. For instance, California's population in 2000 constituted 12.06% of the national population.[7] Thus, under the idea of proportional representation, California should receive 12.06% of the 435 congressional seats, or 52.46 seats. California's "fair share" is therefore 52.46. However, congressional seats must be distributed in integers, and it is therefore impossible to meet a state's "fair share" precisely. As Professor Edelman indicates, "The problem of apportionment amounts to a problem of rounding."[8] The various apportionment methods essentially provide different mechanisms to round the number of a state's representatives to an integer.

There are six primary apportionment methods, each of which is named after the historical figure who first presented the method: Hamilton, Jefferson, Adams, Webster, Hill, and Dean.[9] The methods can be subdivided into two primary categories: the Hamilton Method and the five Divisor Methods.[10] Within the category of Divisor Methods, there is a sub-category based on means.[11] The Hill and Dean methods rely on the geometric and harmonic means, respectively.[12] As mentioned previously, the Constitution does not designate a particular method, and the Hamilton, Jefferson, Webster, and Hill methods all have been used by Congress.[13]

The Supreme Court first examined the constitutionality of the various apportionment methods in the 1992 case, *U.S. Department of Commerce v. Montana*.[14] Montana brought suit based on the 1990 census numbers claiming that the Hill method was unconstitutional. Montana argued that the Hill method, which had been used since

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7. Edelman, *supra* note 1, at 335.

8. *Id.* at 311.

9. MICHAEL L. BALINSKI & H. PEYTON YOUNG, FAIR REPRESENTATION, MEETING THE IDEAL OF ONE PERSON ONE VOTE 60–66 (2d ed. 2001).

10. The genuine divisor methods consist of the Jefferson, Adams, and Webster methods. For a discussion on how to calculate these methods see *id.*

11. *Id.* at 61–63.

12. The harmonic mean of two numbers is their product divided by their average. For instance, to calculate the harmonic mean of 1 and 2: First, 1 is multiplied by 2, which equals 2. Second, 2 (the product) is divided by 1.5 (the average). 2/1.5 = 1.33. The harmonic mean of 1 and 2 is 1.33. The geometric mean is calculated by taking the square root of the product of two numbers. Thus to calculate the geometric mean of 1 and 2, 1 is multiplied by 2, which equals 2. Then take the square root of 2 = 1.414. *Id.* at 62. The way the Dean and Hill method works is that if the "fair share" is greater than the harmonic or geometric means, the quota is rounded up. If the "fair share" is below the harmonic or geometric means, the quota is rounded down. Edelman, *supra* note 1, at 313 n.96.

13. Massachusetts v. Mosbacher, 785 F. Supp. 230, 246–48 (D. Mass. 1992).

14. 503 U.S. 442 (1992).

1941, does not account for the Court's "one-person, one-vote" jurisprudence.[15] Under the Hill method, Montana received one representative, and unsurprisingly, under the Dean method Montana would receive two representatives. The Court held that the Hill method was not unconstitutional and deferred to Congress to determine which method to use for apportionment.[16]

Professor Edelman provides definitive evidence that the Court erred in calculating certain figures. Although Professor Edelman correctly asserts that the Court miscalculated relative *deviation*, as he describes it, the Court should have calculated the relative *difference* in a different manner than that proposed by Professor Edelman or the Court.

III. THE ERROR: "RELATIVE DIFFERENCE" IS NOT NECESSARILY THE SAME THING AS "RELATIVE DEVIATION"

The confusion here likely results from the distinction between terms used in districting and terms that should be used in apportionment. There are three key *districting*[17] terms that are fundamental to this discussion: absolute deviation, relative deviation, and total deviation. Contrast these three districting terms to the two terms used in the *apportionment* context: relative difference and absolute difference. There is a fundamental distinction between districting and apportionment: districting involves *deviation* and apportionment involves *difference*. In footnote 111 of his Article, Professor Edelman claims that the conflation of "relative difference" and "relative deviation" essentially represents a typo by the Court: "The Court uses the terms 'relative difference' rather than relative deviation."[18] According to Professor Edelman, when the Court claims to be calculating "relative difference," it is actually calculating "relative deviation," and in doing so, the Court subsequently miscalculated the "relative deviation."[19] However, I contend that the terms "relative deviation" and "relative difference" can, and should, have independent significance in the districting and apportionment contexts and should be calculated differently.

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15. *Id.* at 446–47.

16. *Id.* at 465.

17. Districting is what takes place within a state when the state draws the boundaries for state or congressional representatives. Apportionment is the process by which congressional representatives are allocated to the states.

18. Edelman, *supra* note 1, at 315 n.111.

19. *Id.*

In the districting context, the states historically have been constrained by two criteria: (1) seats cannot be allocated on the basis of race,[20] and (2) states must adhere to the principle of "one person, one vote," "as nearly as is practicable."[21] This background is important because in state districting cases there is a strict ten percent total deviation benchmark standard. That is, the total deviation in the state cannot exceed ten percent.[22] Or put in other terms, the spread, or difference, between the highest and lowest relative deviations cannot exceed ten percent. To calculate total deviation, one first must determine the ideal district size. This is done by dividing the total population by the number of districts resulting in an "ideal" apportionment. After the ideal district size is calculated, the next step is to measure the relative deviation. Relative deviation is calculated by subtracting the actual district size from the ideal district size and then dividing the resulting number by the ideal district size. To calculate the total deviation, the third step is to look at the highest and lowest relative deviations and calculate the difference, or spread, between the two numbers. (In Table 1 from Professor Edelman's article the highest deviation is 7.11 and the lowest deviation is -4.76; thus, the total deviation is 11.88).[23]

The Court has set limits for total *deviation* and these limits are used to determine the constitutionality of the districting scheme. Total deviation is predicated on first calculating relative deviation. When a state is drawing congressional districts, there are innumerable ways to allocate the seats; thus, calculating relative and total deviations is informative in the districting context.

The apportionment context is an entirely different entity. In the apportionment context, comparisons are not made between different districts, but rather are made between different apportionment methods. So while relative *deviation* is informative in the districting context when comparing the deviation of numerous districts, relative *difference* is informative when comparing the various apportionment methods.

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20. Gomillion v. Lightfoot, 364 U.S. 339 (1960). This Response does not analyze the race constraint. See Grant M. Hayden, *Resolving the Dilemma of Minority Representation*, 92 CAL. L. REV. 1589 (2004), for a good discussion on the topic.

21. Reynolds v. Sims, 377 U.S. 533, 559 (1964) (quoting Wesberry v. Sanders, 376 U.S. 1, 8 (1964)).

22. Connor v. Finch, 431 U.S. 407, 418 (1977).

23. Edelman, *supra* note 1, at 300–06. Courts do not hold congressional districting cases to the ten percent standard, but to a "zero deviation standard." *E.g.,* Karcher v. Daggett, 462 U.S. 725, 727 (1983); Vieth v. Pennsylvania, 241 F. Supp. 2d 478, 483 (M.D. Pa. 2003).

The reason this distinction must be made is that in the districting context, there are innumerable alternative districting schemes, and the districting schemes are measured by their deviations from the ideal size. However, in the apportionment context, the comparisons are not made between infinite districting schemes, but between two alternative apportionment methods. Additionally, unlike districts—which are compared to an "ideal" standard—achieving the ideal standard in apportionment is impossible.[24]

There is historical evidence that "difference" and "deviation" should have independent significance in the apportionment and districting contexts. In the 1920s, Congress commissioned the National Academy of Sciences (NAS) to prepare a mathematical report of the different apportionment methods, and the report indicated that *difference*, as opposed to *deviation*, should be used in apportionment:

> The NAS committee tested equity by applying pair-wise comparisons, a commonly used approach that consists of examining the effects of moving a seat between any pair of states. . . . The measures of inequity are expressed either as absolute *differences* or as relative *differences* in persons per representatives . . . .[25]

Thus, when comparing different apportionment methods, *difference* between the methods, as opposed to *deviations* within the methods, is the benchmark that should control the analysis.

IV. Relative Difference versus Absolute Difference

Acknowledging that the apportionment context involves a different analysis from the districting context, the question faced by the Court in *U.S. Department of Commerce v. Montana* was whether the difference should be measured according to the relative or absolute differences.[26] Professor Edelman argues that the Court chose the Hill apportionment method because it resulted in a lower relative difference than the Dean method.[27]

The justification for choosing relative difference over absolute difference is explained compellingly by Michael L. Balinski and H. Peyton Young in their seminal book on apportionment:

> Why should the relative difference in representation be the proper measure of inequality? Hill observed that if a state has average constituency 100,000, and another 50,000, their absolute inequality is 50,000; whereas, if one has 75,000 and another

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24. For further discussion as to why this task is illusory, see *U.S. Department of Commerce v. Montana*, 503 U.S. 442, 463 (1992).

25. Declaration of Lawrence R. Ernst at 23–24, U.S. Dep't of Commerce v. Montana, 503 U.S. 442 (1992) (No. CV91-22-H-CCL) (emphasis added).

26. Edelman, *supra* note 1, at 317–18.

27. *Id.* at 316.

> 25,000, their absolute inequality is again 50,000, or the same. And yet the inequality in representation seems to be worse in the second case than in the first. For in the first case the state with 50,000 per representative is 100% better off, and in the second case the state with 25,000 per representative is 200% better off.[28]

Balinski and Young then show empirically how the Hill and Dean methods differ in their application to the 1910 census.[29]

Dean

State	Population	Quota	Dean Appt.	Constituency	Relative Difference	Absolute Difference
Massachusetts	3,366,316	15.71	15	224,428		
					19.3	36,273
Florida	752,619	3.512	4	188,155		

Hill

State	Population	Quota	Hill Appt	Constituency	Relative Difference	Absolute Difference
Massachusetts	3,366,316	15.71	16	210,401		
					19.2	40,472
Florida	752,619	3.512	3	250,873		

There is one distinguishing feature about this table: With respect to the Dean and Hill methods, there is only one calculation for relative difference and absolute difference. The reason for this is clear if the distinction between districting and apportionment is understood. As noted previously, relative deviation is used in the districting context to compare the deviation among various districts; the analysis compares the various districts, and therefore, multiple calculations are necessary to compare the numerous districts. However, in the apportionment context the comparison is not between the districts, but between the apportionment methods—the districts are merely tools used to compare the methods.

In the apportionment context, the gravamen of the question is: which apportionment method gives the "better" difference? The follow-up question is whether a person would like to minimize the relative or absolute difference. As noted by Balinski and Young, the Hill method

28. BALINSKI & YOUNG, *supra* note 9, at 48.
29. *Id.* at 49 tbl.6.1.

always minimizes relative difference, and the Dean method always minimizes absolute difference.[30]

Now compare Balinski and Young's model to the Table provided by the Court:[31]

Dean	Average District Size	Absolute Difference From Ideal	Relative Difference from Ideal
Montana (2 Representatives)	401,828	170,638	42.50%
Washington (8 representatives)	610,993	38,257	6.70%
Total Absolute Difference		209,165	

Hill	Average District Size	Absolute Difference From Ideal	Relative Difference from Ideal
Montana (1 Representative)	803,665	231,189	40.40%
Washington (9 Representatives)	543,105	29,361	5.40%
Total Absolute Difference		260,550	

As one can see from the table, the calculation is intrinsically flawed. The Court tried to apply its districting model, calculating relative *deviation*, to the apportionment model, which calculates relative *difference*. The problem is that there should only be one number in the relative difference column: the relative difference between Montana and Washington, not the relative deviation from ideal of the districts within Montana or Washington.

30. *Id.* at 49.
31. *Montana*, 503 U.S. at 462.

The correct table should have been:

Dean

State	Population	Quota*[32]	Dean Appt.	Constituency	Relative Difference	Absolute Difference
Montana	803,655	1.404	2	401,828		
					52.05%	209,165
Washington	4,887,941	8.538	8	610,993		

Hill

State	Population	Quota	Hill Appt	Constituency	Relative Difference	Absolute Difference
Montana	803,665	1.404	1	803,655		
					47.98%	260,550
Washington	4,887,941	8.538	9	543,105		

Relative difference asks *how many times* (the degree to which) Montana is better off than Washington. Thus to calculate relative difference, a comparison must be made between the two states, as opposed to relative deviation, which looks at the deviation from ideal within the states. For instance, using the Dean method, the average district sizes are 401,828 and 610,993 for Montana and Washington, respectively. To calculate the relative difference under the Dean apportionment model, the larger number is divided by the smaller number (610,993/401,828) which results in 1.5205, or expressed as relative difference: 52.05%.

Professor Edelman correctly asserts that the Court computed relative *deviation* incorrectly. Additionally, if the Court was trying to calculate relative *difference*, the Court also miscalculated that number. Indeed, the Court did not calculate relative *difference* or relative *deviation* accurately.

V. A BETTER WAY TO CALCULATE RELATIVE DIFFERENCE

As indicated previously, there is a distinction between relative *deviation* in the districting context and relative *difference* in the apportionment context. The problem is that the Court claimed that it was calculating relative *difference*, but used the calculation for

32. BALINSKI & YOUNG, *supra* note 9, at 48.

relative *deviation*, which (as Professor Edelman points out correctly) it then computed incorrectly.

However, the number the Court claimed to calculate, relative *deviation*, is not informative in the apportionment context. *Deviations* typically are used to quantify differences within a large set of numbers, as in the case of standard deviation or the dispersion of numbers from their expected mean. However, calculating difference connotes a comparison between a smaller set of numbers, often the difference between two numbers. We see that in districting, there are large sets of numbers that include multiple districts and thus, relative deviation is informative because of the large set of possible alternatives. However, in the apportionment context, one compares the marginal difference between representative allocations. Thus, the comparison in apportionment is constrained to a choice between two alternatives: allocating the final seat to one state or another—here Montana or Washington. For this reason, the NAS considered *difference* when making "pair-wise" comparisons.[33]

A mathematical argument against the calculations explained above is that mathematical difference between two numbers implies the subtraction of one number from the other. Thus, relative difference could be used merely to express the absolute difference as a percentage. In essence, this is what Professor Edelman calculated: absolute difference expressed as a percentage.

Professor Edelman calculates the "Total Deviation" to be 36.5% under the Dean method, and 45.5% under the Hill method. Now if one uses the absolute difference resulting from the Hill method, 260,550, and divides it by the ideal size, 572,466, the answer is 45.5%. If one performs the same calculations using the Dean method, one divides the absolute difference, 209,165, by the ideal size, 572,466, and the answer is 36.5%. Thus, following Professor Edelman's line of calculation, it is evident that calculating relative deviation essentially expresses absolute difference as a percentage.

This calculation, therefore, is not informative. Using Professor Edelman's calculations, the apportionment that produces the lowest absolute difference, the Dean method, always will produce the lowest "relative difference." However, the Court must not have been looking for this type of "relative difference" because that "relative difference" is not informative. If the Court could determine unambiguously that a certain calculation produces a lower absolute difference, there would be no need to calculate that difference as a percentage. It may make sense in the districting context when relative deviation is compared to

33. Declaration of Lawrence R. Ernst, *supra* note 25, at 22–24.

the benchmark of ten percent, but there is no similar benchmark in the apportionment context. Thus, calculating relative difference in this manner does not add value, and it contradicts the Court's rule of surplusage. The Court would be calculating a figure that does not have independent or significant meaning.

To give independent and significant meaning to the term "relative difference," the difference must be calculated according to Balinski and Young's method as demonstrated above. This calculation always will result in the Hill method producing lower relative differences and the Dean method producing lower absolute differences.

VI. CONCLUSION

Professor Edelman provides an in depth look at the history of apportionment and correctly asserts that the Court made a mathematical error. However, the calculation could have, and should have, been calculated in a different way than Professor Edelman indicates. Even though the Court miscalculated relative difference, it came to the correct conclusion that the Hill method does result in a lower relative difference than the Dean method. Calculating relative deviation according to the method demonstrated by Balinski and Young would give greater insight into the relative differences of the Dean and Hill apportionment methods and would properly differentiate relative *difference* from relative *deviation*.