10-1972

“Budgetary Constraints and Benefit-Cost Criteria

W. Kip Viscusi

Brief Reports

Budgetary Constraints and Benefit–Cost Criteria

W. KIP VISCUSI

4407 Dannywood Road, Louisville, Kentucky 40220

Abstract. Conventional benefit–cost guidelines are erroneous, owing to their failure to recognize the realities of policy making. From appropriate consideration of budgetary constraints, the interdependence of projects, and the influence of project selection on future budgets the conclusion is that project selection should not follow crude rules of thumb, such as the order of project benefit–cost ratios. By means of a dynamic Lagrangian multiplier model the maximum economic advantage principle for the political context is derived. This principle implies that an optimal project schedule does not require that all marginal benefit–cost ratios be equal. For example, if a project increases the economic desirability of other projects, its own economic value increases. Moreover, a project increasing future budgetary allocations takes on increased economic desirability, since one of the project benefits equals the sum of the increased budgetary allotments, as weighted by the appropriate budgetary shadow premiums.

A standard tenet in the theory of benefit–cost analysis is that project size should be increased until the marginal benefit from expanding a project no longer exceeds the marginal cost of adding project segments. The conventional corollary to this tenet is that, to obtain the optimal scale of development, the marginal benefit–cost ratios of all projects should be 1.0.

Government officials appear to have been swayed by the apparent logic of this analysis. For example, the Department of the Interior (unpublished document, 1959) claims that the optimal size of a reclamation project is attained 'by omitting from the plan those segments yielding less benefits than costs, and adding segments producing more benefits than costs.'

However, the theoretical framework on which this planning procedure is based assumes no budgetary constraints. The first major treatment of the effect of budgetary constraints was that of *Eckstein* [1958], who focused on the implications of budgetary constraints in a one-period model. Eckstein's static system demonstrated that for optimal project selection the ratio of discounted marginal net benefits to discounted marginal federal costs must equal $1 + \lambda$, where λ is the shadow premium of capital due to a budgetary constraint. *Marglin* [1962] extended Eckstein's conclusions to a multiperiod model in which all projects selected for funding in any year u must have a marginal benefit–cost ratio equal to $1 + \lambda_u$, where λ_u is the shadow premium for the budget in year u.

RECOGNITION OF PROJECT INTERDEPENDENCE IN A DYNAMIC MODEL

Previous analyses of either the static or the dynamic planning problem have not incorporated the facts that benefit and cost streams of water resource projects are interdependent and that the selection of a particular project for construction alters the available set of projects for future years. However, because it is technologically infeasible for two dams to be located in the same position, the set of available possibilities is affected by the projects constructed. Similarly the benefit and cost figures for one dam will be altered significantly if another dam is located 10 miles upstream from it. Although more subtle examples of these phenomena exist, the two major kinds of interdependence, economic and technological, are contained in the preceding example.

Look at the implications for an agency's investment planning of assuming project interdependence in a simple dynamic model based on the following eight assumptions:

1. Accurate data inputs: The benefit–cost estimates are accurate reflections of the value of each project to the public.

2. Discrete time periods: The decision maker is concerned with the selection of the optimal mix for the current period and the next h periods, where the length of each period is 1 year. (Although my analysis will consider investment problems over time, it will deal with discrete time periods and not with a system incorporating continuous benefit–cost streams. This approach does not modify the thrust of my conclusions and is in keeping with current federal practices.)

3. Constant discount rate: The discount rate i, used to discount the benefit–cost streams, is constant.

4. Known future budgetary allotments: The vector of budgetary allotments for an agency's expenditures in the current period and the next h periods is known. (Unlike *Marglin* [1962] I do not assume that this budgetary vector allows the eventual construction of all projects whose benefit–cost ratios are >1.0. In practice, agencies like the U.S. Bureau of Reclamation receive appropriations for individual projects instead of funds in the annual lump sum form that we are using in this analysis. In addition, my analysis assumes that the amount of funds available for expenditures in the years considered does not depend on the projects selected between now and the time considered.)

5. Project costs: All private costs are negative benefits and netted out in determining the benefit figure for a project. All federal project costs are placed in a fund such that the present value of such costs discounted back to the year of project authorization is the relevant measure of the contribution to the exhaustion of the budgetary constraint. In the event that an additional segment of a project is funded in a subsequent year, these funds will be treated as agency expenditures for the year of this appropriation rather than for the year in which the project was begun. For any given year the federal capital costs of the projects plus the associated discounted (to the year of appropriation) federal operation and maintenance costs cannot exceed the budgetary allotment for that year. Funds cannot be carried over to subsequent years, and project benefits are unavailable for future use.

6. Project life: Each project k is in operation until T_k years after the beginning of project construction. All benefit and cost streams from project k are included in the figures for the T_k years of operation.

7. Interdependent benefit–cost streams: The benefit and cost streams of water resource projects need not be independent. In mathematical terms the general formulations for the benefit and cost streams of a project k constructed in year u are not functions solely of the scale of project k funded in year u. Rather, the various benefit and cost functions are now described by functions of $r \cdot (h + 1)$ variables.

8. Interacting variable set of available projects: The scale of a particular project to be constructed in any given year is selected from a set X of available projects. However, the set X is modified by the construction of projects in years previous to the year being considered, by the other projects selected for construction in the year being considered, and by commitments to construct projects and project segments in future years.

The variables used are as follows:

- A_u, agency's allotment of federal funds in year u;
- B_{tku}, undiscounted benefit streams (net of private costs) accruing in year t from the parts of project k authorized in year u;
- C_{tku}, undiscounted federal costs in the tth year of operation of segments of project k adopted in year u;
- h, last year of decision making considered relevant to the analysis;
- i, discount rate;
- k, subscript indicating the kth project, equal to $1, 2, \cdots, r$;
- r, number of possible water resource projects available for the agency's construction;
- t, year of project operation (number of years after project adoption), equal to $0, 1, \cdots, T_k$ for any project k;
- u, year of project or project segment adoption, equal to $0, 1, \cdots, h$;
- W_u, change in social welfare due to projects adopted in year u;
- x_{ku}, scale variable indicating the scale of development of project k funded in year u.

Note that assumption 7 implies that the benefit and cost functions take on the general form in which $B_{tku} = B_{tku}(x_{10}, x_{11}, x_{12}, \cdots, x_{1h}, x_{20},$

$x_{21}, \cdots, x_{2h}, \cdots, x_{r0}, \cdots, x_{rh})$ and $C_{tku} = C_{tku}(x_{10}, x_{11}, x_{12}, \cdots, x_{1h}, x_{20}, x_{21}, \cdots, x_{2h}, \cdots, x_{r0}, \cdots, x_{rh})$. Rather than always denote the benefit and cost functions as functions of $r \cdot (h + 1)$ variables, I will simplify my notation by using the functional symbols on the left-hand side of the preceding equations (e.g., B_{tku}) instead of the more complete forms on the right-hand side of these equations.

The task of maximizing the increase in the welfare of society is achieved by maximizing

$$\sum_{u=0}^{h} W_u = \sum_{u=0}^{h} \sum_{k=1}^{r} \sum_{t=0}^{T_k} (B_{tku} - C_{tku}) \cdot (1 + i)^{-t-u} \quad (1)$$

subject to the constraint that

$$\sum_{k=1}^{r} \sum_{t=0}^{T_k} C_{tku} \cdot (1+i)^{-t-u} \leq A_u \cdot (1 + i)^{-u} \quad (2)$$

$$\forall \quad u = 0, 1, \cdots, h$$

We must therefore maximize the Lagrangian expression

$$L = \sum_{u=0}^{h} \left\{ W_u - \lambda_u \left[\sum_{k=1}^{r} \sum_{t=0}^{T_k} C_{tku} \cdot (1 + i)^{-t-u} - A_u \cdot (1 + i)^{-u} \right] \right\} \quad (3)$$

where λ_u is the Lagrangian multiplier for year u.

To derive the first order conditions necessary for maximization, we must take the partial derivatives of this expression with respect to each λ_u as well as each x_{ku}. However, the partial derivatives with respect to the x_{ku} cannot be conveniently represented by using the present summation conventions, since each benefit and cost function is a function of all the x_{ku} and not just of the x_{ku} of the particular project being considered. To ease the resulting complications, denote the notational subscripts of the x_{ku} in the partial derivatives by k' and u' instead of by k and u. Let k' include the integers 1–r as k did. Similarly let u' include the integers 0–h as u did. There is no substantive difference in my analysis except that I take the partial derivatives of the benefit and cost functions (with subscripts including k and u) with respect to the $x_{k'u'}$, where k' and u' need not equal k and u, respectively.

Using the Lagrangian technique, we find that the necessary conditions for welfare maximization are

$$\sum_{k=1}^{r} \sum_{u=0}^{h} \left\{ \sum_{t=0}^{T_k} \frac{\partial B_{tku}}{\partial x_{k'u'}} \cdot (1 + i)^{-t-u} \cdot \left[\sum_{t=0}^{T_k} \frac{\partial C_{tku}}{\partial x_{k'u'}} \cdot (1 + i)^{-t-u} \right]^{-1} = 1 + \lambda_u \right\} \quad (4)$$

$$\forall \quad u' = 0, 1, \cdots, h$$

$$\forall \quad k' = 1, 2, \cdots, r$$

Through fairly simple algebraic manipulation this result can be expressed in two main ways. Using verbal equations to express the necessary marginal conditions for maximization, we find that each project segment k' should be expanded in year u' until (1) the sum of the marginal net discounted benefits due to $x_{k'u'}$ and accruing to all projects constructed in the $h + 1$ years of funding equals the sum over all r projects and all $h + 1$ years considered of [the discounted marginal federal costs due to $x_{k'u'}$ and accruing to each segment x_{ku}] times [the relevant $1 + \lambda_u$, i.e., the budgetary shadow price of x_{ku}] or until (2) the sum over all projects in all $h + 1$ years considered of [the discounted net marginal benefits due to $x_{k'u'}$ and accruing to each segment x_{ku} constructed] minus {[the marginal discounted federal costs due to $x_{k'u'}$ and accruing to each project segment x_{ku} constructed in the $h + 1$ years considered] times [the relevant $1 + \lambda_u$, i.e., the budgetary shadow price for x_{ku}]} equals 0.

Thus the net marginal benefits from each project must just equal the marginal federal costs (weighted for the budgetary constraint) for optimality. The major difference from other analyses is that we must sum these marginal benefit and cost figures over all projects, owing to the interdependence of project outputs. Thus the external benefits and costs of a project in relation to those of other projects must be considered in determining the optimal stream of investments. For mutually exclusive projects the project that maximizes the difference between the sum of the net discounted benefits and the discounted federal costs (weighted by the appropriate shadow prices) should probably be selected. The actual selection of projects cannot be done on a project-by-project basis, however. Rather, the entire investment schedule must be selected to maximize the well-being of society.

Perhaps the most important conclusion is that it is no longer true that the marginal benefit–cost ratios of all projects should equal the appropriate budgetary shadow price for the year of construction. If a project creates particularly favorable opportunities for water resource development or has other beneficial external effects, it should be constructed. The necessary marginal conditions for the maximization of the welfare of society are given by (4).

The time pattern of investments is important not only because of shifts in demand for project outputs and varying budgetary constraints over time but also because of project interdependence. For example, the construction of projects in a particular area (e.g., the development of the Missouri River basin) or along a common waterway (e.g., the dams on the Snake and Colorado rivers) makes it imperative to recognize the proper sequence for construction. Failure to do so will result in a nonoptimal investment pattern.

I label this principle the maximum economic advantage principle, since we must look at the interdependence of project impacts as well as the comparative advantage of particular projects for different time periods. This principle should not be confused with the common practice of maximizing the difference between calculated project benefits and costs. The maximum economic advantage principle differs in that it calls for weighting all project costs with budgetary shadow prices.

MODEL EXTENDED TO INCLUDE AN ENDOGENOUS BUDGETARY CONSTRAINT

In the preceding section I assumed that the amount of agency funds to be allocated in the $h + 1$ years considered was exogenous to the system. Assumption 4 of the model required that the budgetary allotment vector be known and fixed. However, in reality, funds depend on the availability of productive projects and similar factors. One of the reasons frequently cited for the downward trend in appropriations for new projects is that there are only a finite number of locations for dams and similar facilities. This project interdependence was the problem focused on in the preceding section. The sequence of project construction and the performance of an agency's projects also affect future appropriations. If a project is built that tends to augment the benefits of related projects in a system of projects, such as those in the Missouri River basin system, the effect will be similar to that of increasing the economic desirability of investment opportunities; i.e., appropriations will probably rise. The performance of projects is also important for promoting a favorable attitude in Congress toward such development.

To incorporate the various influences, view the budgetary allotment in any period as a function of an agency's past projects as well as of those planned for future construction. Assumption 4 now becomes:

4′. Variable budgetary allotments: The amount of funds available for the agency's expenditures in any year u is given by a well-defined function A_u, known to the decision makers. Each A_u is a function of the projects funded in past years and those selected for current and future construction. We continue to depart from Marglin's approach, since we are still assuming that this budgetary vector need not allow the eventual construction of all projects whose benefit–cost ratios are >1.0.

Although other influences, such as the general state of the economy, also affect appropriations, they have not been incorporated into the model, since they would complicate the results without giving any compensating insights. We have, however, focused on the interaction between political and economic factors in this model. The budgetary allotments are no longer taken as an exogenous component produced by some vague political force. Rather, through appropriate choice of projects this constraint is subject to modification for the reasons previously indicated.

The notation is the same as it was in the previous section except that now $A_u = A_u(x_{10}, x_{11}, \cdots, x_{1h}, \cdots, x_{20}, x_{21}, \cdots, x_{2h}, \cdots, x_{r0}, \cdots, x_{rh}) \; \forall \; u = 0, 1, \cdots, h$. To simplify the exposition, this function will be indicated by A_u rather than by the right-hand side of the preceding equation. The problem now is to maximize (1) subject to (2), where A_u takes on the new significance outlined above. We get the result of this constrained maximization problem by using Lagrangian multipliers. The necessary conditions for maximization are

$$\sum_{k=1}^{r} \sum_{u=0}^{h} \left\{ \sum_{t=0}^{T_k} \frac{\partial B_{tku}}{\partial x_{k'u'}} = (1 + \lambda_u) \sum_{t=0}^{T_k} \left[\frac{\partial C_{tku}}{\partial x_{k'u'}} \cdot (1 + i)^{-t-u} \right] - \lambda_u \frac{\partial A_u}{\partial x_{k'u'}} \cdot (1 + i)^{-u} \right\} \quad (5)$$

$$\forall \quad k' = 1, 2, \cdots, r$$

$$\forall \quad u' = 0, 1, \cdots, h$$

Thus, for maximization of the welfare of society, each project segment $x_{k'u'}$ must be expanded until the sum over all projects constructed in the $h + 1$ years of funding of the marginal net discounted benefits (discounted benefits less discounted private costs) due to project segment $x_{k'u'}$ and accruing to each project constructed equals {[the discounted marginal federal capital costs to each project x_{ku} due to $x_{k'u'}$] times [the budgetary shadow price for x_{ku}, i.e., $1 + \lambda_u$]} minus {[the budgetary shadow premium λ_u] times [the change in budgetary allotments in year u due to $x_{k'u'}$, discounted appropriately]}.

The marginal conditions for determining project scale are the same as those in the previous section except for the addition of the last term. In the event that a segment $x_{k'u'}$ increases funds in any year (i.e., $\partial A_u/x_{k'u'} > 0$), the last term in the expression above, i.e., $\lambda_u \cdot (\partial A_u/\partial x_{k'u'}) \cdot (1 + i)^{-u}$, will be positive, as could occur when a particular project segment results in increased opportunities for an agency's project development in a river basin. If the sum over all $h + 1$ years of these budgetary increments weighted by the budgetary shadow premiums is positive, projects should be expanded more than they would be if the budgetary allotments were taken as given. Similarly, if the sum over all $h + 1$ years of the weighted budgetary increments is negative, projects should be expanded less than they would be if the budgetary allotments were fixed exogenously.

Similarly the criterion for selecting a project from a mutually exclusive group or for selecting the proper amount of stage construction is altered. Again, to maximize the well-being of society, the project schedule that maximizes the difference between net benefits and federal costs, as weighted by the budgetary shadow prices, should be selected. However, the determination of this result is dependent on the effects of project appropriations as well as on the interdependence of project effects and the comparative advantage of projects in different time periods. This principle will be termed the maximum economic advantage principle for the political context to distinguish it from the principle derived in the preceding section. The necessary marginal conditions for the attainment of this maximum are given by (5).

CONCLUSION

For the most part the standard principles of project evaluation are undermined by the appropriate recognition of budgetary constraints. Policy makers are economically and politically justified in refusing to follow the conventional recommendation that projects be constructed in the order of their benefit–cost ratios. The maximum economic advantage principle for the political context implies that, if the adoption of a project with a low benefit–cost ratio is likely to increase the desirability of other projects or to increase future budgetary allotments, public officials are on firm economic ground in following their political inclinations. Furthermore, the interdependence of projects combines with the influence of project selection on budgetary constraints to justify abandoning the current project-by-project method of timing and evaluation in favor of selecting entire project schedules.

Note that actual budgetary constraints are probably not as rigid as those specified in my models. Sometimes such constraints can be overridden for a finite political cost that may take the form of decreased appropriations in future years. However, the essential policy conclusions of the preceding models should not be affected by more flexible budgetary constraints. Such a modification would complicate the determination of the budgetary shadow prices without adding any conceptual insights. As a result I have focused on the implications of well-defined budgetary limits. The essential importance of all the models I have developed is to demonstrate the necessity of recognizing the political context of economic analyses. Failure to recognize this environment and its effect on economic analyses will perpetuate the use of inappropriate principles for allocating public funds.

Acknowledgment. I am especially indebted to Professor Richard J. Zeckhauser for many helpful comments. This paper is adapted from chapter 2 of my unpublished honors thesis 'Selected Topics in Public Expenditure Analysis' (April 1971), which received the 1971 Allyn A. Young prize at Harvard College.

REFERENCES

Eckstein, O., *Water-Resource Development,* Harvard University Press, Cambridge, Mass., 1958.

Marglin, S. A., Economic factors affecting system design, in *Design of Water-Resource Systems,* edited by A. Maass et al., pp. 159–225, Harvard University Press, Cambridge, Mass., 1962.

(Manuscript received November 4, 1971; revised April 6, 1972.)