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THE AMENDING OF ARTICLES 23 AND 27 OF THE UNITED NATIONS CHARTER: A MATHEMATICAL ANALYSIS

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I. Intoduction

From the time of the San Francisco Conference, the composition of the Security Council and its voting procedure was most severely criticized. The basic criticism had been the veto power of the five permanent members on substantive resolutions.[1] This resulted in a long and vigorous political struggle on the part of the non-veto members of the Organization to amend the Charter in order to increase their voting strength in the Security Council.[2] When the changes on membership and voting procedure came into force on August 31, 1965, a great victory was claimed.[3] This case of the United Nations is intriguing and deserves attention of legal scholars, mainly because the proponents for the amendment to the Charter had no knowledge of intricacy in the relationship between its unique procedural formality of voting mechanism

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1. For official accounts of this criticism see, 11 UNITED NATIONS INFORMATION ORGANIZATION, DOCUMENTS OF THE UNITED NATIONS CONFERENCE ON INTERNATIONAL ORGANIZATION, SAN FRANCISCO, 1945 (1945).

2. See G.A. Res. 1919, 18 U.N. GAOR Supp. 15, U.N. Doc. A/5515 (1963). See generally 2 REPERTORY OF PRACTICE OF UNITED NATIONS ORGANS 3-14 (1955) [hereinafter cited as REPERTORY OF PRACTICE]; 5 REPERTORY OF PRACTICE 401; 1 REPERTORY OF PRACTICE 243-47, 269-76 (Supp. 1, 1958); 2 REPERTORY OF PRACTICE 277-82, 307-14 (Supp. 2, 1964). For a narrative study of the development see, Schwelb, Amendments to Articles 23, 27, and 61 of the Charter of the United Nations, 59 AM. J. INT'L L. 834-56 (1965).

3. Prior to the change, the membership of the Security Council consisted of 5 permanent and 6 non-permanent members. As amended it consists of 5 permanent and 10 non-permanent members. The amended Article 27 provides that decisions of the Council on non-procedural matters shall be made by an affirmative vote of 9 members (formerly 7), including the concurring votes of the 5 permanent members of the Security Council.

and the distribution of voting power as was "concealed" in the Charter. The purpose of this article is not to discuss juridical points but to analyze this concealed knowledge which should have been known in the course of amending the Charter. The analysis will demonstrate that the so-called political victory turned out to be an empirical defeat and it will examine power distributions under two alternative plans for future amendments to the Charter.

II. Defining the a priori Voting Power

The concept of power has been widely used by social scientists from the Greek period; however, it is still an imprecisely defined concept.[4] For the purpose of this paper, the concept is defined from a limited standpoint, based on one of the properties of power of a committee system. In a committee, which decides by voting, members have certain portions of voting strength. This voting strength, excluding other properties of power whatever they may be, is called the *a priori* voting power.

The formula measuring the *a priori* voting power as applied in this paper was developed in 1954.[5] According to Shubik and and Shapley, "the voting power of an individual member [of a committee] depends on the chance he has of being critical to the success of a winning coalition."[6] In a voting committee

4. *See generally* Dahl, *Power*, 12 INT'L ENCYC. SOC. SCI. 405-15 (1968) (contains an especially helpful bibliography).

5. Shapley & Shubik, *A Method for Evaluating the Distribution of Power in a Committee System*, 48 AM. POL. SCI. REV. 787-92 (1954). The Shapley-Shubik study is the only method available for measuring the *a priori* power of a committee system. Some attempts have been made to measure other properties of "power." *See generally* Riker, *Some Ambiguities in the Notion of Power*, 58 AM. POL. SCI. REV. 341-49 (1964).

6. Shapley & Shubik, *supra* note 5, at 787.

whose rules of voting prescribe what proportion of votes constitutes a winning proportion, each voting member has a certain probability of casting the last required vote for a winning coalition. Hence the voting power of each committee member depends on the ratio of the number of possible times each member may cast the pivotal vote in an ordered permutation to the number of ordered permutations possible.[7] Suppose the k members are ordered $m_1, m_2, \ldots, m_k$ according to how likely they are to vote for a measure. If the prescribed rules of the committee for the winning coalition is q, then a winning coalition must contain $m_1, m_2, \ldots, m_q$. Therefore, if the set $m_1, m_2, \ldots, m_q$ is a winning coalition, then the m_q is the pivotal position. To state this reasoning mathematically, we quote William Riker's expression as follows:

$$P_i = \frac{m(i)}{n!}$$

> where P is the power to determine outcomes in a voting body for a participant, i, in a set of participants: 1, 2, . . . , n where m(i) is the number of times i is in the pivotal position and where pivotal position is defined thus: when the rules define q vote as winning,
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> $$\frac{n+1}{2} \leq q \leq n \qquad \text{or} \qquad \frac{n}{2} + 1 \leq q \leq n,$$
>
> the pivot position is the qth position in an ordered sequence of votes.[8]

Let us illustrate. Suppose k=3. Suppose also, each of the three-member committee has one vote and the winning coalition is by majority. In the sequence of m_1, m_2, m_3, the pivotal vote

7. For a mathematical proof of this indexing scheme see, Shapley, A Value For N-Person Games, 28 ANNALS OF MATH. STUDY 307-17 (1953). For an elementary treatment of permutation see, J. KEMENY, J. SNELL, & G. THOMPSON, INTRODUCTION TO FINITE MATHEMATICS chs. 3, 4 (2d ed. 1966) (the Shapley-Shubik index is described at 79-82; applications of the index may be found at 113-15). Cf. Brams & O'Leary, An Axiomatic Model of Voting Bodies, 64 AM. POL. SCI. REV. 449-70 (1970).

8. Riker, supra note 5, at 341.

is the second position, (m_2). Thus there are 2! ways each member may become the pivot. Therefore, the a priori voting power of each member of this committee is

$$\frac{2!}{3!} = \frac{1}{3}$$ [9]

III. The Distribution of Voting Power in the Security Council

We now apply this principle to the Security Council of the United Nations. According to the original rules prescribed in Article 27 of the Charter, the resolutions of a substantive nature required seven majority votes, including the five permanent members. In other words, a winning coalition was required to contain $m_1, m_2, \ldots, m_7$ and a losing coalition was $m_1, m_2, \ldots, m_{7-1}$. Therefore, the pivotal position is m_7. Hence a non-permanent member can yield its voting power when it casts the 7th positive vote, following the Five. Computation of the voting power of each member of the Security Council is not so simple (for a detailed explanation, see Appendix). First, it is necessary to find out the number of ways a non-permanent member can be arranged to become a pivot. The number of arrangements are

$$\binom{5}{1} .6! \ .4!$$ [10]

for the following reasons. We denote the Big Five and the Small Six by $B_1, B_2, B_3, B_4, B_5; S_1, S_2, S_3, S_4, S_5, S_6$. The permutation of one S preceding the pivot is $\binom{5}{1}.6!$. Next the permutation of four other S's is 4!. Therefore, there are exactly $\binom{5}{1}.6! \ .4!$ alignments in which one S becomes the pivot. Since $P_i = \frac{m(i)}{n!}$, the a priori voting power of a Six is

$$\frac{\binom{5}{1} .6! \ .4!}{11!} = \frac{1}{462} \quad \text{or} \quad 0.002164.$$

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9. E.g., the number of arrangement is: $m_1, m_2, m_3; m_1, m_3, m_2; m_2, m_1, m_3; m_2, m_3, m_1; m_3, m_1, m_2; m_3, m_2, m_1$. The second vote being the pivot, each number has power of $\frac{2}{6} = \frac{1}{3}$.

10. $\binom{5}{1} = \frac{5!}{1! \ (5-1)!} = 5$ is the number of combinations of taking one object at a time out of five objects. In general, $\binom{n}{r} = \frac{n!}{r! \ (n-r)!}$.

Because the sum total of the powers of the members must be $\frac{k!}{k!} = 1$ and because the sum total power of the Six is $\frac{6}{462}$ or $\frac{1}{77}$, the sum total power of the Five is $1 - \frac{1}{77} = \frac{76}{77}$. Consequently, the voting power of a Five is $\frac{76}{77} \div 5 = \frac{76}{385}$ or 0.197403.

Now, let us compute the distribution of power of the Security Council under the amended Articles 23 and 27. Under the new rules, the Security Council consists of fifteen members of which ten are non-permanent members and five are veto-holding permanent members. The substantive resolution requires nine votes, including the five permanent members. The voting power of a non-permanent member is now thus:

$$\frac{\binom{9}{3}.8!\ .6!}{15!} = \frac{4}{2145} \quad \text{or} \quad 0.001865$$

IV. Findings and Analysis

This analysis reveals some interesting points and suggests some interesting theoretical implications. First, the *a priori* voting power of each of the non-permanent members has now been reduced from 0.002164 to 0.001865 or decline of 0.000299. But the voting power of the non-permanent members as a group has increased from 0.012984 to 0.018648 or gain of 0.005664. This gain, however, is meaningless because they do not vote collectively. Of course, it should be pointed out that the extent of self-defeat by loosing the voting power of 0.000299 is minute. But there are two significant aspects of this finding: one is that had this analysis been known to the United Nations, the politics of amending the Charter would have been entirely different (better alternative courses are discussed below), and the other is that this analysis can be applied to any future changes of the Security Council. Of the former aspect it seems safe to conclude that the members of the United Nations had never carefully analyzed the result of the change, but rather intuitively and falsely assumed that an enlargement of the non-permanent membership would weaken the voting power of the permanent members.[11]

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11. It is submitted that the mathematical analysis of voting power discussed in the text has simply been unknown

Secondly, these power indices reveal the distribution of the original *a priori* voting power between the Five and the Six at the ratio of 98.7% : 1.3% respectively. Individually the ratio between a Five and a Six is 90 : 1. These findings cast doubt on the intuitive notion that the non-permanent members have "bargaining power" in the politics of the Security Council. It has been argued that the Great Powers could not dictate to the Security Council because numerically the non-permanent members constitute a majority and because the coalition of the Great Powers requires two additional votes of support from the non-permanent members. As analyzed above, however, the so-called "bargaining power" is negligible.

Thirdly, had the amendment to Article 27 required ten majority instead of nine (including the permanent members), it would have been better politics because of the following reasons: 1) the voting power would have been more equally distributed, and 2) the voting powers of the non-permanent members would have been increased approximately twice. For example, under this ten majority rule, the distribution of voting power would be thus:

A) The voting power of a Ten is

$$\frac{\binom{9}{4}.9!.5!}{15!} = \frac{126}{30030} \quad \text{or} \quad 0.004195$$

B) The voting power of the Ten is

$$\frac{126}{30030} \times 10 = \frac{1260}{30030} \quad \text{or} \quad 0.041958$$

C) The voting power of the Five is

$$1 - \frac{1260}{30030} = \frac{28770}{30030} \quad \text{or } 0.958042$$

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to the political world in general. An interesting quantitative study somewhat related to this comment might be mentioned here. In his study of the migration of French deputies from one party to another in 1953 and 1954, William Riker found no evidence supporting the contention that the legislators carefully analyzed effects on their voting powers resulting from the change of party affiliation. Riker, A Test of the Adequacy of the Power Index, 4 BEHAVIORAL SCI. 120-31 (1959).

D) The voting power of a Five is

$$\frac{28770}{30030} \div 5 = \frac{28770}{150150} \quad \text{or} \quad 0.191608$$

Fourthly, still a better alternative would have been the thirty-nine member draft resolution submitted on November 3, 1969.[12] This resolution called for the change of the Security Council membership from eleven to thirteen and fro the change of voting procedure for the adoption of a substantive matter by the Security Council from seven to eight, including the permanent members. If the majority requirement would have been nine instead of eight, it would have met the objective of the non-veto members far more satisfactorily than under the original rules and procedures of the Security Council. The following table illustrates this point.

Comparison of Non-Permanent Members' Voting Power

	The Original System: 11 = 5 + 6 with 7 majority	The Amended System: 15 = 5 + 10 with 9 majority	Alternative System I: 15 = 5 + 10 with 10 majority	Alternative System II: 13 = 5 + 8 with 9 majority
Voting power of One Non-permanent Member	0.002164	0.001865	0.004195	0.005439
Voting Power of All Non-permanent Members	0.012984	0.018648	0.041958	0.043512

Finally, this analysis suggests an interesting implication for the legal community. It is submitted that perhaps legal scholarship should endeavor in developing certain theories on the relationship between the variations of voting body as well as constitution writing.

12. See U.N. GAOR, Special Pol. Comm. Res. 25, U.N. Doc. A/SPC/L.25 and Add. 1-3 (1969).

APPENDIX

THE FORMULA FOR COMPUTING THE VOTING POWER OF THE SECURITY COUNCIL

$n = 11.$	Number of members.
$v = 5.$	Number of permanent members.
$n - v = 6.$	Number of non-permanent members.
$n - v - 1 = 5.$	Number of non-permanent members except the pivot, k.
$m = 7.$	Number of majority rule required.
$m - v - 1 = 1.$	Number of non-permanent members preceding k.
$\binom{n - v - 1}{m - v - 1} = \binom{5}{1} = 5.$	Number of ways choosing m-v-1 out of n-v-1.
$(m - 1)! = 6!$	Number of permutation of m - 1 members before k which include five permanent members.
$(n-m)! = 4!$	Number of permutations of members after k.
$\dfrac{\binom{n - v - 1}{m - v - 1} \cdot (m - 1)! \cdot (n - m)!}{n!}$	Formula for Power Index of a Six.