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The Myth of the Condorcet Winner

Paul H. Edelman

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There is consensus among legal scholars that, when choosing among multiple alternatives, the Condorcet winner, should it exist, is the preferred option. In this essay, I will refute that claim, both normatively and positively.

I. INTRODUCTION

In the area of social choice it is difficult to achieve a consensus on very much. Within the somewhat more restricted area of applying social choice to law (as well as political science) there has been one issue, though, for which a consensus has been achieved—when choosing among multiple alternatives, if there is one alternative that is preferred to every other alternative in a pairwise comparison, a so-called *Condorcet winner,* then that alternative should be selected.[1]

It is a little surprising that this position has come to dominate. The battle over this assertion is at least 300 years old, going back to arguments between Marquis de Condorcet and Jean-Charles de

* Professor of Mathematics and Law, Vanderbilt University, paul.edelman@vanderbilt.edu. I would like to thank Ed Cheng, Saul Levmore, Rob Mikos, Suzanna Sherry, and Allan Wiseman for helpful comments.

[1] Bernard Grofman and Guillermo Owen, *Condorcet Models, Avenues for Future Research*, in B. Grofman and G. Owen, eds, *Information Pooling and Group Decision Making: Proceedings of the Second University of California, Irvine Conference on Political Economy* 97 (JAI 1986); Saul Levmore, *Parliamentary Law, Majority Decision-making, and the Voting Paradox,* 75 Va L Rev 971, 995 (1989); Maxwell Stearns and Todd Zywicki, *Public Choice Concepts and Applications in Law* 103 (West 2009).

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Borda in late eighteenth-century France.[2] The current preference for a Condorcet winner seems to be based on some combination of normative claims about democratic fairness as well positive claims about the stability of the choice. At any rate, "Scholars have identified the Condorcet criterion as an important benchmark in evaluating the decision-making competence of institutions."[3]

But, as so often happens, while legal scholars[4] have considered the matter settled, work of social choice scholars has shed quite a different light on the merits of a Condorcet winner. The primary purpose of this article is to bring attention to these results and show how they undercut the accepted wisdom. Taken together, these developments cast considerable doubt on the assumption that the concept of the Condorcet winner has any explanatory power in analyzing the behavior of legislatures.

This essay will proceed as follows: In Section II I present the requisite background and explain the political and legal significance of the Condorcet winner. In the next section, I will exhibit a large class of examples in which the Condorcet winner is, as a normative matter, likely not the best choice. These examples, which were originally developed by Saari[5] and discussed at length by Balinski and Laraki,[6] are constructed using Condorcet components, sets of preferences which cycle in pairwise majority voting.

Section IV raises a theoretical objection to the importance of the Condorcet winner. I will discuss a number of theoretical results which demonstrate that any social choice procedure that always selects a Condorcet winner, if one is available, necessarily fails to satisfy a number of consistency conditions. These consistency conditions lead to, among other pathologies, the "no-show" paradox where voters can be worse off if they cast a ballot than if they fail to vote at all. I will argue that this paradox is particularly problematic in the context of a deliberative body such as Congress. Section V closes the essay with a brief conclusion.

[2] George Szpiro, *Numbers Rule: The Vexing Mathematics of Democracy, from Plato to the Present* chs 5, 6 (Princeton 2010).

[3] Maxwell Stearns, *An Introduction to Social Choice,* in Daniel A. Farber and Anne Joseph O'Connell, eds, *Research Handbook on Public Choice and Public Law* 90 (Elgar 2010).

[4] And many political scientists as well, to be fair.

[5] D.G. Saari, *Chaotic Elections! A Mathematician Looks at Voting* (Am Mathematical Socy 2001).

[6] Michel Balinski and Rida Laraki, *Majority Judgment, Measuring Ranking, and Electing* 74 (MIT 2010).

II. BACKGROUND AND LEGAL CONTEXT

In this section, I will introduce some basic terminology from social choice and introduce the idea of the Condorcet winner.[7] Then I will discuss both the political and legal significance of the Condorcet winner.

The standard model of social choice assumes that a set of voters, which we will label {1, 2, . . . , n} are confronted with a number of alternatives, which we will denote {A, B, . . . , C}. We will assume that each voter has a preference order over the alternatives. For example, we will indicate that voter 1 prefers A to both C and B, and prefers C to B, by saying that 1's preference order is A > C > B. For simplicity we will assume that every voter has strict preferences, that is, there are no ties among her preferences. A collection of voters and their preference lists will be called a *profile*. The central problem in social choice is preference aggregation—how to go from a profile to a single societal choice.[8] A method that takes as input the individual preferences of the voters and outputs the societal choice will be called a *social choice function*.

A fundamental conundrum for social choice is the problem of cycling. For example, suppose we have the following profile:

8	B > C > A
10	C > A > B
6	A > B > C

where by 8: B > C > A I mean that there are eight voters with preference B > C > A. With these preferences there is no clearly preferred alternative: fourteen of the twenty-four people have B > C, sixteen have A > B, and eighteen have C > A. Such a situation is known as a Condorcet cycle. Cyclic preferences raise the concern that any choice is unstable, that is, it can be subsequently replaced by a preferred one in a pairwise-majority vote.

Although the existence of Condorcet cycles has been known for centuries, it was only after the fundamental work of Arrow,[9] and

[7] Readers who would like a more thorough introduction can see Wulf Gaertner, *A Primer in Social Choice Theory* (Oxford 2006).

[8] Actually one might also ask to go from the profile to a single societal preference order instead of picking a single winner. While this may seem like an equivalent problem the connection between picking a societal winner and picking a societal order is very subtle. See H.P. Young, *Optimal Ranking and Choice from Pairwise Comparisons*, in Grofman and Owen, *Condorcet Models* at 113 (cited in note 1).

[9] Kenneth Arrow, *Social Choice and Individual Values* (2d ed Yale 1963).

later that of Gibbard and Satterthwaite,[10] that the scope of the challenge to democratic legitimacy became clear. The short version of this is that any reasonable method by which a legislature turns the preferences of its members into a decision is subject to manipulation (for example, by agenda control or introduction of new alternatives) or cycling.

There are many ways that one might aggregate the preferences of the voters. Since the fundamental work of Arrow, the approach to choosing among the various methods has been axiomatic. That is, one decides a priori what properties a method should satisfy and then see which methods exhibit the desirable properties. So, for example, one often requires that a method exhibit the Pareto property: if every voter prefers A to B, then society should not pick B. Most of the common social choice methods satisfy this property, although not all of them.[11] The fundamental insight of Arrow is that there are no social choice functions that are consistent with a particular small set of very natural properties.[12]

For this essay the property of particular interest is the Condorcet criterion. We call an alternative a *Condorcet winner* if it would beat every other alternative in a pairwise majority vote. That is, alternative A is a Condorcet winner if, given any other alternative X, more people prefer A to X than vice versa. It is important to note that a Condorcet winner may not exist, as is demonstrated by our earlier example. A social choice function is said to satisfy the Condorcet criterion if, whenever there is a Condorcet winner, the social choice function selects it as the winner. Such methods will be called *Condorcet consistent*.

It has become an article of faith among legal academics (and some political scientists) that Condorcet-consistent methods are the most desirable ones in the context of political decision making.[13] The Condorcet criterion is viewed as the natural generalization of majority rule in the two-alternative context. Majority rule is attractive from both a normative and a positive standpoint, but it really can only

[10] A. Gibbard, *Manipulation of Voting Schemes,* 41 Econometrica 587 (1973); M.A. Satterthwaite, *Strategy-Proofness and Arrow's Conditions: Existence and Correspondence Theorems for Voting Procedures and Social Welfare Functions,* 10 J Econ Theory 1 (1975).

[11] In particular, the method of sequential pairs under a fixed ordering may select a Pareto inferior alternative. See Alan D. Taylor, *Mathematics and Politics: Strategy, Voting, Power, and Proof* 114 § 5.5 (Springer 1995).

[12] For one explicit statement (and there are many variations) see id at 251.

[13] See the references cited in notes 1 and 3. See also Daniel B. Rodriguez, *State Constitutional Failure,* 2011 U Ill L Rev 1243, 1253 (2011); Jonathan Remy Nash, *A Context-Sensitive Voting Protocol Paradigm for Multimember Courts*, 56 Stan L Rev 75, 96 (2003).

be applied when the choice is between alternatives for which there will always be a majority winner. If more alternatives are available then there will typically be no majority winner. If there is a Condorcet winner then it might be considered "the majority will" even if it is not "the majority's will,"[14] a term reserved for a true majority choice.[15] It is this connection to "majority rule" that makes the Condorcet criterion so appealing. But like many appealing axioms, such as the Pareto axiom already referred to, there may be hidden problems that undermine its desirability.

If we are concerned about stability of legislative decisions we may also find the Condorcet winner a desirable choice. Since it beats every other alternative in a pairwise vote, it is difficult to destabilize the selection once it has been made. More generally we might think that a rational legislature would structure its rules so that the choice of legislation would be Condorcet consistent. That at least would ensure avoidance of Condorcet cycles whenever it is possible to do so. Levmore has suggested that the procedural rules of Congress do, in fact, have that effect.[16]

The debate over how legislatures convert the preferences of its members into a single choice has implications for how courts should interpret statutes. One conclusion from Arrow's theorem and related results might be that "legislative intent is an oxymoron because the relationship between the individual preferences of legislators and the collective product of the legislative body is not discoverable."[17] "The whole idea that statutes have purposes or embody policies becomes quite problematic, since the content of the statute simply reflects the haphazard effect of strategic behavior and procedural rules."[18] As a consequence some judges have argued that it makes no sense to consider the intent of the statute and any interpretive issues should be decided solely on the basis of the written statute itself.

There are a number of responses to this conclusion, some within public choice itself,[19] but one way to try and reclaim the idea of

[14] Gerald Kramer, *Some Procedural Aspects of Majority Rule*, in J. Roland Pennock and John W. Chapman, eds, *Nomos XVIII: Due Process* 268 (NYU 1977).

[15] This distinction between "the majority will" and "the majority's will" strikes me as interesting and underexamined. A Condorcet winner beats all other alternatives because of different majorities coalescing to defeat each alternative rather than a fixed majority that prefers one alternative to all others.

[16] Saul Levmore, *Public Choice Defended,* 72 U Chi L Rev 777, 781 (2005).

[17] Jerry Mashaw, *Public Law and Public Choice: Critique and Rapprochement*, in Farber and O'Connell, eds, *Research Handbook on Public Choice and Public Law* at 19, 39 (cited in note 3).

[18] Daniel A. Farber and Philip P. Frickey, *Law and Public Choice* 41 (1991).

[19] For a good summary of these arguments, see id at 38.

legislative intent is to argue that legislative procedures should be, and are, Condorcet consistent. If we believed that the legislature always chooses a Condorcet winner when it is available, it would bolster our confidence that the institution is behaving as rationally as possible, given the theoretical constraints of Arrow's theorem, and thus perhaps be more confident in identifying intent behind their legislative actions.

In this article I wish to cast doubt upon this method of reclaiming legislative intent.[20] In the next section I will illustrate why the Condorcet winner, when available, may not, as a normative matter, be the obviously correct choice among multiple alternatives. In the following selection I will show that even if we think the Condorcet winner should always be chosen there are severe disadvantages inherent in any procedure that would guarantee such an outcome. The result is to say that we should neither privilege the Condorcet winner nor attempt to ensure its selection.

There have been other critiques of the Condorcet winner condition. There is the obvious concern that a Condorcet winner need not exist. Of course this concern goes back several hundred years. Another criticism is that the Condorcet winner does not take into account the intensity of preference.[21] But none of the standard social choice procedures allow for this sort of accounting,[22] and so the criticisms I present are more general.

III. IS THE CONDORCET WINNER THE BEST CHOICE?

In this section I will give an example to show that the Condorcet winner should not be the presumptive choice among multiple alternatives. It is based on the idea that collections of preferences can cancel each other out and effectively be removed from the analysis.

To start, consider this very special preference profile:

10	B > C > A
10	C > A > B
10	A > B > C

[20] To be clear, I am not arguing that the notion of legislative intent is incoherent, only that advancing the Condorcet criterion as a way of reclaiming it is incorrect.

[21] Stearns, *An Introduction to Social Choice* at 92 (cited in note 3).

[22] Except, perhaps, for the allowance of log-rolling in legislation. See Dennis C. Mueller, *Public Choice II* 82 (Cambridge 1989).

This profile gives rise to a Condorcet cycle, B > C > A, but this cycle has the property that every pairwise vote is numerically exactly the same—the winner gets twenty votes and the loser gets ten. So the voting outcomes are completely symmetric in the alternatives—there is no way to distinguish one from another. Given this outcome, the only reasonable conclusion is that all three alternatives are tied.[23] If we had to decide a winner we might resort to flipping a coin, or some other random tie-breaking rule. Otherwise the alternatives are indistinguishable in terms of their support. I will refer to situations such as these, where there are equal numbers of voters with cyclic preferences which create indistinguishable alternatives, as *Condorcet components*.

The significance of a Condorcet component is that it represents a tie among the alternatives. Since all of the alternatives receive exactly the same amount of support we should view them as being identical and we have no principled way to decide among them. So, if a subset of voters form a Condorcet component, and hence cannot decide among the alternatives, then a reasonable approach would be to let the rest of the voters' wishes prevail, if they are able to come to a consensus. It is this idea that is at the heart of our example.

We can now construct examples of preferences where, as a normative matter, the Condorcet winner is arguably not the "correct" winner. Consider the following set of preferences:[24]

30	A > B > C
1	A > C > B
29	B > A > C
10	B > C > A
10	C > A > B
1	C > B > A

[23] I am making the assumption that we wish to treat all the voters identically—in technical terms the social choice function is anonymous. If we allow some voters to have more clout than others, then there may well be a natural unique "winner."

[24] This example is attributed to Condorcet who, ironically, used it to undermine the voting method proposed by Borda. See Szpiro, *Numbers Rule* at 90 (cited in note 2). Of course Condorcet approached this example from the position that a Condorcet winner, if it exists, is presumptively the correct choice. More recently Saari has used this example to illustrate the difficulties of the Condorcet criterion. See Saari, *Chaotic Elections!* at 74 (cited at note 5). I will follow the presentation of Balinski and Laraki, *Majority Judgment* at 74 (cited in note 6).

In this collection of preferences, A is the Condorcet winner—A is preferred to B by forty-one of the eighty-one voters, and A is preferred to C by sixty of them. So the Condorcet criterion would assert that A should be the social choice.

But let us think about this in another way. These preferences contain the Condorcet component

10	B > C > A
10	C > A > B
10	A > B > C

(the last group of ten is a subset of the thirty voters with those preferences.) As we already noted, the result of a Condorcet component is that for these thirty voters there is just a tie among all three alternatives. Since from the perspective of these thirty voters there is a tie among all the alternatives, we should leave the decision to the rest of the voters. So let us remove those thirty votes from consideration.[25] There is another Condorcet component that can be removed as well:

1	A > C > B
1	C > B > A
1	B > A > C

But after removing these 30 + 3 voters whose preferences result in a collective tie among the alternatives we are left with the preferences

20	A > B > C
28	B > A > C

For this set of preferences it is rather clear that B should be the winner, since it is preferred over A by a score of 28 to 20 and C is clearly the third choice. But remember, it is A, not B, which is the Condorcet winner.

In the next section I will show that the phenomenon illustrated by this example is inherent to any social choice method that satis-

[25] The idea of removing voters from consideration who collectively tied is commonplace. In a regular majority vote between two alternatives a couple who would vote in opposite ways may mutually agree not to vote since it is easier and would not affect the outcome.

fies the Condorcet criterion. But for now the point to be made is that it is not self-evident that a Condorcet winner should always be the social choice.

Before moving to that discussion it is worth noting that often scholars will advance a practical argument in favor of a Condorcet winner. If a Condorcet winner is available but not chosen, then a majority coalition exists that will be opposed to the current choice and favor the Condorcet winner. Such a situation is inherently unstable. It leads, the argument goes, to either a change to the Condorcet winner or a disgruntled majority that feels disenfranchised.

How seriously we should take this concern depends on a number of things. The first is whether the various legislative procedures would allow for the majority to force a vote between the Condorcet winner and the currently chosen option. Most legislatures make it difficult to reconsider issues, so there may be few opportunities for a majority coalition to overturn the original choice.

One might also wonder whether the level of unhappiness of the majority, if they are unable to alter the decision, would be sufficient to undermine the government. The legislative procedures might be sufficiently murky that the majority may be unsure as to the real strength of their position. The costs of attempting to overturn the decision may be sufficiently high, or the benefits sufficiently diffuse, to make it inefficient even to try to overturn the decision, even if they were certain that they were in the majority.

Another factor in assessing this argument is the question, if this kind of instability or majority displeasure truly manifested itself, what would we expect to see in situations in which there was no Condorcet winner at all? In those cases one would expect to see continuous cycling of the chosen alternatives and considerable unhappiness among the shifting majorities that have lost. But it is, in fact, very difficult to identify actual instances of cycling in legislative decision making,[26] and so the fears of this kind of instability would appear to be overblown.

IV. CONDORCET WINNERS AND THE NO-SHOW PARADOX

In the previous section I gave an example to show that a Condorcet winner need not be the presumptive social choice. The basis of the

[26] "Tullock noted that the theory that predicts cyclical majorities stands in sharp contrast to reality and asks why stability is observed in legislatures, although instability is predicted by innumerable chaos theorems." Thomas Stratmann, *Logrolling*, in Dennis C. Mueller, ed, *Perspectives on Public Choice: A Handbook* 332 (Cambridge 1997).

argument was that when groups of voters essentially negate each other (because they form a Condorcet component) the deciding voters may prefer an alternative other than the Condorcet winner. In this section I will show how this example illustrates an inherent problem in methods that always select Condorcet winners.

Before proceeding further, however, it is worth noting that the problem illustrated in the previous section extends beyond just the normative claim that the Condorcet winner is not the obviously correct social choice. It actually can be extended to show that any social choice function that is Condorcet consistent can be destabilized by the addition of a Condorcet component, as we now illustrate.

A social choice function is said to *cancel properly* if the social choice remains unchanged when the voters are supplemented by adding a Condorcet component. That is, if the social choice is alternative A for a particular profile, then changing the profile by adding a set of voters which forms a Condorcet component will result in the same social choice. As an example, suppose a social choice function selects option A given the profile

8	A > B > C
12	B > A > C
15	C > A > B

If the social choice function cancels properly, then A will still be the social choice for the profile

18	A > B > C
10	B > C > A
12	B > A > C
25	C > A> B

because the latter profile differs from the former by the addition of the Condorcet component

10	A > B > C
10	B > C > A
10	C > A > B

The property of canceling properly is, unfortunately, inconsistent with the Condorcet criterion, as was shown by Balinski and Laraki.[27]

***Theorem*: There is no social choice function that is Condorcet consistent and cancels properly**.

This theorem shows that if a social choice function will always select a Condorcet winner, if one is available, then the result might be changed by adding voters who form a Condorcet component.

The reader might think that the argument in favor of canceling properly is not compelling. Is that the worst that can happen if we require Condorcet consistency? Well, no. There is an inherently related property that is considerably more problematic, especially in the context of legislative decisions. That is the property of *join consistency.*

Imagine that the House of Representatives has to choose among three alternatives A, B, and C for some policy and they are using a certain social choice function to do so. Suppose that using that function the California delegation would choose alternative C. In addition suppose that the rest of the members of the House, using the same choice function, would also select C. What should happen if the whole House makes a decision? One would naturally assume that C should be the choice. That is the property of join consistency: If two separate electorates each choose the same alternative, then when the two electorates act as one, they should make the same choice.

Unfortunately, join consistency is not compatible with Condorcet consistency, as this theorem of H. Peyton Young demonstrates:[28]

***Theorem*: No social choice function that is Condorcet consistent is also join consistent**.

This theorem seriously undermines the desirability of a Condorcet-consistent social choice function. A Condorcet-consistent function must, necessarily, lead to the following possibility: Suppose both the California delegation and the remaining members of the House (each deciding separately) prefer alternative C. If the social choice function is Condorcet consistent then it is necessarily join inconsistent and thus, acting as one, the House might choose alternative B instead. This puts the California delegation in an untenable position. They could advance their preferences by NOT appearing at the vote. This is referred to as a *no-show paradox*.

[27] Balinski and Laraki, *Majority Judgment* at 77 (cited in note 6).
[28] See id.

An example will help illustrate this. Suppose the method of election is to first pair off A against B and then that winner pairs off against C. Let group X be the eleven voters with preferences given by

3	A > B > C
3	C > A > B
5	B > C > A

In this election A beats B 6 to 5 and then C beats A 8 to 5, so the group choice is C.

Now suppose another group Y, also with eleven voters, has the preferences

5	B > A > C
1	A > C > B
5	C > B > A

In their election B beats A 10 to 1 and then C beats B 6 to 5 also resulting in C being the choice.

But what happens if X and Y vote together? In that combined vote B beats A 15 to 7, but then B also beats C 13 to 9, resulting in B being the winner! So while each group separately prefers C in a unified vote they choose B.

One might think that the no-show effect is solely of academic concern and unlikely to arise in real life. While I do not have an example of it arising in an election employing a Condorcet-consistent method (which is what is suggested by the previous theorem), there are plausible claims that it manifested itself in a mayoral election in Burlington, Vermont, in 2009.[29] In a four-candidate race for mayor, in an election using instant runoff voting, which is not Condorcet consistent,[30] the Condorcet winner actually came in third. There was a group of voters who ranked the actual winner third, but had they not voted at all, their second-ranked candidate would have won. So the outcome would have been better for them had they just

[29] See http://www.rangevoting.org/Burlington.html. But compare http://www.fairvote.org/more-on-warren-smiths-and-anthony-gierzynskis-flawed-analysis/; compare http://www.rangevoting.org/BurlResponses.html.

[30] Instant runoff voting is sometimes called Hare voting. See Taylor, *Mathematics and Politics* at 100 (cited in note 11).

stayed home. Thus, the no-show paradox is one that does actually arise, and if the election uses a Condorcet-consistent voting method there will always be the concern that it will manifest itself.

As just illustrated, employing a method that is join inconsistent could lead to groups not participating in a decision as way to advance their own interests. This undermines the goal of participation as a key component of deliberative groups. Yet it is a problem inherent to social choice functions that always choose a Condorcet winner. Thus, as a theoretical matter, methods that are Condorcet consistent are put into question.

The failure of any Condorcet-consistent method to satisfy join-consistency was previously noted by William Riker.[31] He refers to it as a "very serious defect" and claims that the problem in fact manifested itself during the time that state legislatures were selecting senators.[32] He summarizes the issue this way: "Failing consistency, then, majoritarian [Condorcet-consistent] methods are at least impractical as well, perhaps, as unfair."[33] These serious reservations seem to have been missed by a number of authors because he routinely is cited in support of the position the importance of the Condorcet criterion.[34]

V. CONCLUSION

The goal of this Article is to undermine the significance of the Condorcet winner to public choice. Despite its recurring invocation as the obvious choice, there are both theoretical and practical reasons why we should be skeptical of its importance. Yes, there is a superficial appeal of "majority rule" when advocating for a Condorcet winner, but if there is any lesson to be learned from Arrow at all it is that superficial appeals rarely withstand scrutiny. There are, alas, no self-evident correct alternatives even in the situations where a Condorcet winner exists.

[31] William H. Riker, *Liberalism Against Populism: A Confrontation between the Theory of Democracy and the Theory of Social Choice* 102 (Freeman 1982). He calls the condition "consistency."

[32] Id. I should note that I am not entirely sold on his example, but that is a different story.

[33] Id at 105. Later he says, "We should think of the methods, I believe, simply as convenient ways of doing business, useful but flawed. This gives them all a place in the world, but it makes none of them sacrosanct." Id at 113.

[34] See for, example, Stearns and Zywicki, *Public Choice Concepts* at 103 (cited in note 1); Levmore, 72 U Chi L Rev at n 69 (cited in note 16).