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Utility Functions That Depend on Health Status: Estimates and Economic Implications

By W. KIP VISCUSI AND WILLIAM N. EVANS*

Taylor's series and logarithmic estimates of health state-dependent utility functions both imply that job injuries reduce one's utility and marginal utility of income, thus rejecting the monetary loss equivalent formulation. Injury valuations have unitary income elasticity, and the valuation of non-incremental risk changes and effects of base risks follow economic predictions. (JEL 851,026,913)

In the basic von Neumann-Morgenstern framework, individual utility depends on a single attribute—one's wealth. In some contexts, the character of the lottery payoffs may be so sweeping that it transforms the utility function. Consider, for example, a utility function for a risk-averse individual so that utility increases with wealth, but at a diminishing rate. If one were to treat death as being tantamount to a drop in income, then one would obtain the unreasonable result that death boosts the marginal utility of income. This implausible result highlights the fallacy of treating death and other severe health effects as monetary equivalents.

Robert Eisner and Robert H. Strotz (1961) first noted this class of difficulties in their analysis of flight insurance. They suggested that a bequest function is a more appropriate formulation of the utility function after one's death. Modification of the standard utility theory approach to recognize the complications posed by other forms of state dependence has not posed any insurmountable difficulties, as the theory of state-dependent utility is now well developed.[1]

Economists have applied the state-dependent approach to a diverse set of economic problems involving irreplaceable effects, product safety, and accidents.[2] By far the most widespread use of this formulation is with respect to state-dependent variations with individual health status. Richard J. Zeckhauser (1970, 1973) and Kenneth J. Arrow (1974) developed analyses of health care and health insurance decisions in which the utility functions for good and ill health may assume quite different shapes. These formulations led to an overhaul of the economic analysis of the optimal structure of health insurance.

In particular, let there be n discrete states of the world indexed by $j = 1, \ldots, n$. Each state has an associated health level, $health_j$, and income level Y_j. One can then write individual expected utility EU as the sum of the utilities in each health state weighted by the probability of s_j that that health state j occurs, or

$$EU = \sum_{j=1}^{n} s_j U(health_j, Y_j),$$

as in Charles E. Phelps (1973) and Arrow

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[1]See, for example, Arrow (1964), Jack Z. Hirschleifer (1970), Ralph L. Keeney and Howard Raiffa (1976), and Edi Karni (1985).

[2]See Zeckhauser (1970, 1973), Arrow (1974), Philip J. Cook and Daniel A. Graham (1977), Spence (1977), Viscusi (1978, 1979), Joseph Pliskin, Donald S. Shepard, and Milton C. Weinstein (1980), Daniel A. Graham (1981), George W. Torrance (1986), Steven M. Shavell (1987), Viscusi and Michael J. Moore (1987), and Victor R. Fuchs and Richard J. Zeckhauser (1987).

(1974). Ideally, one would like to obtain a continuous measure of health capital to analyze explicitly how individual health status alters the structure of utility functions and to explore the economic implications of health. Since our empirical analysis includes only two health states—good health and a post-injury state—a continuous measure of health cannot be constructed. Instead, we subsume the role of health into a state-dependent utility function as in Arrow (1974), or

$$EU = \sum_{j=1}^{n} s_j U_j(Y_j).$$

More specifically, let the notation $U(Y)$ denote the utility function in good health and $V(Y)$ denote the utility function in the post-injury state. This differing notation denotes that there is a health state-dependent component of a stable preference relationship that we cannot estimate explicitly.

Two assumptions are pivotal. First, for any given level of income, one's overall level of utility is greater when in good health than in ill health, or $U(Y) > V(Y)$. This assumption is not controversial; nor does it distinguish the health state approach from earlier models that treated adverse health effects as being equivalent to financial losses. Second, to make any judgments about the extent of the optimal insurance one must make an assessment with respect to the influence of one's state on the marginal utility of income, for any income level. Optimal insurance coverage when there is actuarially fair insurance available will equate the marginal utility of income in each health state (for example, see Zeckhauser (1970, 1973), Arrow (1974), A. Michael Spence (1977), W. Kip Viscusi (1979)).[3] If ill health does not alter the marginal utility of income, for any given income level, then full insurance is optimal. If ill health lowers (raises) the marginal utility of income for any given income level, less (more) than full income insurance is desirable. Thus, the relative magnitudes of $U'(Y)$

and $V'(Y)$ are key empirical parameters. The assumptions one makes about the shape of the utility function govern the fundamental aspects of all of the economic results derived with such models.

In the extreme case of one's death, it is not controversial to assume that one's marginal utility declines after the adverse health effect. For other health outcomes, the justification for assuming a drop in marginal utility is less clearcut. There is no theoretical basis for determining the shape of the utility function in these instances.

In Section I we describe the set of data used to estimate state-dependent utility functions. We will use two empirical approaches. First, Section II imposes no functional form restrictions on the utility function other than a Taylor's series expansion. This unrestricted approach provides tests of the two key assumptions of the state-dependent approach—whether utility is greater in good health or ill health and whether the marginal utility of income is boosted by or reduced by adverse health outcomes. In Section III we impose a specific functional form on the utility function (a logarithmic utility function) and then estimate the utility function in each of the two health states. Section IV uses these results to address a variety of key, but previously unresolved issues, including: the income elasticity of the implicit value of an injury, the valuation of non-incremental changes in risk, changes in risk-dollar tradeoffs with a change in the base level of risk, and the optimal rate of replacement of worker earnings through workers' compensation insurance. Many of these findings are of substantial, independent economic interest.

I. Sample Description

Although there is a considerable literature on wage-risk tradeoffs, estimating individual utility functions is not feasible with standard sets of survey data. Figure 1 makes the source of the difficulty apparent. Let ABC be the frontier of offered wage-risk combinations available in the market. The worker selects the optimal job B from this frontier, where his locus of constant expected utility EU is tangent to the wage of opportunities frontier.

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[3]Although the assumption of actuarial fairness is clearly unrealistic (i.e., it assumes the insurance provides a free service with no administrative costs), it is a frequent reference point in theoretical analyses.
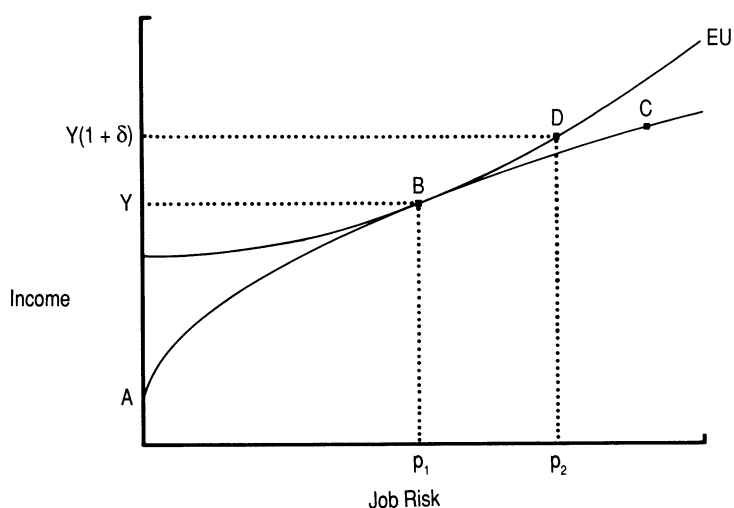
FIGURE 1. THE MARKET OFFER CURVE AND THE WORKER'S EXPECTED UTILITY
LOCUS

Hedonic wage studies of compensating differentials for job risks involve estimation of the average rate of tradeoff for the equilibrium set of expected utility-wage offer tangencies observed in the market. The linear wage equation imposes a constant risk-dollar tradeoff, and a semilogarithmic formulation makes the tradeoff risk-dependent. Market data can only provide evidence regarding the slope of the observed tangencies with the frontier *ABC*. One cannot make any inferences regarding the shape of the individual worker utility functions except with respect to the rate of tradeoff at tangency with the opportunities locus.

We will follow an alternative approach of augmenting market data with reservation wage data obtained in response to different risk levels. In particular, we utilize the 1982 chemical worker survey by W. Kip Viscusi and Charles J. O'Connor (1984).[4] That analysis was primarily concerned with the economic implications of chemical labeling,

whereas this paper is concerned with the utilization of the wage and risk information to estimate worker utility functions.

Table 1 summarizes the characteristics of the chemical worker sample. The personal characteristic variables included information on the worker's age (AGE), sex (MALE dummy variable-d.v.), marital status (MARRIED d.v.), number of children (KIDS), race (BLACK d.v.), years of experience at the firm (TENURE), and education (EDUC).

The survey also elicited the worker's pay for the period that was most meaningful for that particular occupational class. To promote comparability, these statistics have been converted to weekly, after-tax earnings, using average federal and state tax rates for the worker's income class and family status (i.e., marital status and number of dependents).[5] The workers in the sample averaged about $18,000 per year (1982 dollars) in after-tax income.

The key job attribute is the worker's perceived probability of an accident on his job, which is denoted by p_i, $i = 1, 2$, where the

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[4]More specifically, we will utilize the subsample of workers analyzed in Section III of Viscusi and O'Connor (1984). These workers experienced an increase in their job risk. Workers who were randomly assigned to the risk decrease experimental cell were not asked a reservation wage question.

[5]The tax adjustments were made using information provided in The Commerce Clearing House, *State Tax Handbook*, and *U.S. Master Tax Guide*.

TABLE 1—SAMPLE CHARACTERISTICS: MEANS AND STANDARD DEVIATIONS

Variable	Means (Std. Deviation)
AGE (in Years)	38.3
	(11.8)
MALE (0 – 1 Sex Dummy Variable d.v.)	0.56
	(0.50)
KIDS (Number of Children)	1.30
	(1.53)
BLACK (0 – 1 Race d.v.)	0.06
	(0.23)
TENURE (Years of Experience at Firm)	7.77
	(6.97)
EDUC (Years of Education)	14.3
	(3.37)
p_1 (Prior Probability of Accident)	0.084
	(0.055)
p_2 (Posterior Probability of Accident)	0.249
	(0.091)
Y (Weekly before Tax Earnings)	392.13
	(161.52)
δ (Percent Wage Differential)	0.173
	(0.150)
r_1 (Fraction Earnings Replacement by Workers' Compensation on Job 1)	0.637 (0.077)
r_2 (Fraction Earnings Replacement by Workers' Compensation on Job 2)	0.615 (0.093)
t_1 (Average Tax Rate on Job 1)	0.124
	(0.047)
t_2 (Average Tax Rate of Job 2)	0.141
	(0.052)
Sample Size	249

subscript 1 pertains to the pre-labeling situation and the subscript 2 pertains to the post-labeling situation. Workers assessed this probability using a linear risk scale on which there was indication of the average level of the risk for the entire private sector. Workers marked on the scale their assessed job risk with respect to this standardized injury scale, thus providing a risk metric scaled in terms of an annual job risk, that is, the assessed annual probabilities of injury are in the interval [0,1].

The risk metric is equivalent to the U.S. Bureau of Labor Statistics (BLS) private sector injury and illness rate. The risk assessments were 10 percent greater than the private sector average risk for that period and were 46 percent larger than the average chemical industry accident rate. Since BLS accident statistics do not capture the longer term health risks in the chemical industry, this pattern is quite reasonable.

The workers were randomly assigned to one of four different chemical labeling groups—asbestos, TNT, sodium bicarbonate, and chloroacetophenone (an industrial chemical that causes tearing). The workers were told that the chemical would replace the chemical with which they currently worked. Thus, there would be a change in the chemical used rather than a change in the labeling of the chemical with which the individual currently worked. Respondents then assessed the posterior risk, p_2, which is roughly triple the prior risk, Only workers who reported an increased risk assessment are included in the sample analyzed in this paper, since it was only for this group that reservation wage information was obtained. Almost all workers who reported a risk decrease were shown a label for sodium bicarbonate, which was the experimental treatment that corresponded to elimination of the chemical hazards.

The risk scale established a standardized reference lottery that the worker could use in assessing the risk equivalent of his job before and after seeing a warning label. Ideally, both the severity and duration of the reference injury should be the same, where the scale is used to establish differences in probabilities. Although this was the intent of the survey design, the significant differences in injury severity across the label treatment groups may have affected worker responses. In particular, the empirical properties of the responses by workers in the asbestos and TNT label groups are similar, but the behavior implied by the chloroacetophenone group is somewhat different. Since chloroacetophenone leads to only temporary eye irritation, and the other two chemicals pose risks of death, this difference is consistent with the character of the injuries. We will provide empirical estimates for each labeling subsample as well as for the full sample to explore differences across chemicals.

For workers who assessed their job risk as being greater, the survey ascertained the percentage wage increases δ needed to compensate the worker for the increased risk. The average δ value was just under 20 percent. Since the respondents were told that the results would be used for a doctoral dissertation at an institution in a different state and would not be disclosed to their employers, there was no apparent incentive for them to overstate their reservation wage. Moreover, the implicit value of a statistical injury reported in Viscusi and O'Connor (1984) and in Section IV of this paper are not excessively large and are in line with the literature.[6] Section III examines the effect on the estimates of potential response biases.

In terms of Figure 1, the survey first ascertained the information associated with point B—the base risk p_1 and the associated

weekly earnings Y. It then altered the risk to a level assessed by the worker as being p_2, with an associated weekly earnings of $Y(1 + \delta)$, which is point D. Thus, the survey includes information with respect to two points, B and D, on a constant expected utility locus EU where the expected utility EU_1 on the initial job equals the expected utility EU_2 after the wage and risk increase. The starting point and the post-labeling point D differ across workers so that in effect we observe 249 different pairs of points along 249 different utility functions. In contrast, the most that can be accomplished using observed market wage-risk data for this sample is to estimate ABC using the 249 points B. Since the survey generates only one equation, $EU_1 = EU_2$, we are not able to identify the shape of both U and V. However, we are able to generate relationships between the two utility functions, such as differences and ratios.

The final variable needed to complete the formulation of the payoffs in each state is the level of workers' compensation benefits after an injury. Since these benefits are not taxed, for comparability the income in the healthy state is in after-tax terms. The earnings replacement rate variable is based on the state benefit formulas for temporary total disability and the characteristics of the individual respondent. The benefit calculation takes into account the worker's income, benefit ceilings and floors, and the dependence of benefits on family characteristics.[7] The average replacement rate at the initial job (r_1) and for the experimental job (r_2) are both about two-thirds.

Let U denote the utility of wealth in good health, and let V denote the utility of money after a job injury. Then a wage increase that equates the expected utility that the worker obtains from his initial job and the transformed job satisfies

$$(1) \quad (1 - p_1)U(Y(1 - t_1)) + p_1 V(Y r_1)$$
$$= (1 - p_2)U(Y(1 + \delta)(1 - t_2))$$
$$+ p_2 V(Y(1 + \delta)r_2).$$

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[6] The chemical worker sample yields rates of tradeoff that imply a value of $10,000–$20,000 per statistical injury reduced. If there were upward response bias, these estimates should exceed comparable values obtained using market data and hedonic wage equations. However, the estimates in the literature tend to be somewhat greater, as they cluster in the $20,000–$30,000 range. See the survey by Viscusi (1986) and the recent estimates by Viscusi and Moore (1987).

[7] The reasonableness of this approach to capturing empirically the role of workers' compensation is discussed in Viscusi and Moore (1987).

The worker reports his base earnings Y, his required wage increase δ, and his prior and posterior risk assessments, p_1 and p_2. Information regarding the worker's income level is used to construct the tax variables, t_1 and t_2, and the workers' compensation replacement rates, r_1 and r_2. The formulation in equation (1) takes into account the favorable tax treatment of workers' compensation benefits, as taxes only affect earnings in the good health state.

The principal empirical test of whether the state-dependent approach is valid is whether

$$(2) \qquad U(Z) > V(Z)$$

and

$$(3) \qquad U'(Z) > V'(Z),$$

for identical income levels Z in each state. Inequality (2) will be satisfied by both a health state and a monetary loss equivalent model since in each case ill health lowers welfare. The distinctive condition is defined by inequality (3). Under the state-dependent approach, inequality (3) could be in either direction, although the sign in inequality (3) is the more frequent assumption. With a monetary loss equivalent approach, an injury will boost the marginal utility of income for the usual risk-averse preferences, leading to a reversal of inequality (3). If inequality (3) is satisfied, we can reject the monetary equivalent model and the class of health in state models that do not alter the utility function in the manner indicated by inequalities (2) and (3).

II. Estimates with Unrestricted Functional Forms

Ideally, one would like to estimate equation (1) without imposing any restrictions on the shapes of U and V. However, with information on two particular points along the constant expected utility locus, one cannot estimate two different nonlinear functions with available data.

Two approaches are feasible. First, one can estimate specific features of the U and V functions without imposing functional form

restrictions on their shape, as in this section. Second, one can impose constraints on the shapes that U and V can take, as in Section IV. These two different estimation approaches provide a robustness check on the results.

A. First-Order Taylor's Series

The procedure that we adopt in this section is to construct a first-order Taylor's series approximation of utility functions in each health state. The second-order Taylor's series terms, which we will explore in Section IIB, were not statistically significant.

For each of the utility functions, we will use the same level of income as the point of expansion, where this level is Y, the weekly before-tax income. From the definition of the Taylor's series, we generate the following approximations to utility:

$$(4a) \quad U(Y(1-t_1)) \cong U(Y)$$
$$+ \{ Y(1-t_1) - Y \} U'(Y),$$

$$(4b) \quad V(Yr_1) \cong V(Y) + \{ Yr_1 - Y \} V'(Y),$$

$$(4c) \quad U(Y(1+\delta)(1-t_2)) \cong U(Y)$$
$$+ \{ Y(1+\delta)(1-t_2) - Y \} U'(Y),$$

and

$$(4d) \quad V(Y(1+\delta)r_2) = V(Y)$$
$$+ (Y(1+\delta)r_2 - Y)V'(Y).$$

After substituting the values of (4a)–(4d) into equation (1), we obtain

$$(5) \quad (p_2 - p_1)(U(Y) - V(Y))$$
$$= ((1-p_1)t_1 - (1-p_2)(t_2 + \delta t_2 - \delta))$$
$$\times YU'(Y) + (p_1(1-r_1)$$
$$- p_2(1-r_2-r_2\delta))YV'(Y).$$

All of the variables in equation (5) are known except for those parameters involving the

utility functions for each state. It is these terms that will be estimated.

Let

(6a) $\quad \beta_1 = U(Y) - V(Y),$

(6b) $\quad \beta_2 = U'(Y),$

and

(6c) $\quad \beta_3 = V'(Y).$

The dependent variable will be the percentage wage compensation δ that the worker requires to face the increased risk. This empirical structure follows that of the questionnaire, as the survey asked workers how much additional compensation they required to work with the new chemical. Although workers could respond either in absolute or percentage terms, in each case it was the wage premium that was elicited. We can consider δ as the dependent variable since its value is conditioned on knowing all other variables —namely Y, p_1, and p_2.

Inserting the values for β_1, β_2, and β_3 from equation (6) into equation (5) and solving for the endogenous value δ yields

(7) $\quad \delta = \left[\dfrac{(H_1\beta_2 + H_2\beta_3)Y - (p_2 - p_1)\beta_1}{\{(1 - p_2)(t_2 - 1)\beta_2 - p_2 r_2 \beta_3\}Y} \right]$

$\qquad + \varepsilon,$

where

(8a) $\quad H_1 = (1 - p_1)t_1 - (1 - p_2)t_2,$

and

(8b) $\quad H_2 = p_1(1 - r_1) - p_2(1 - r_2).$

Introducing subscripts to denote the ith individual, we have thus hypothesized an empirical relationship of the form

(9) $\quad \delta_i = f(X_i, \beta) + \varepsilon_i,$

where $f(\cdot)$ is a nonlinear function capturing the bracketed term on the right-hand side of equation (7), X_i is a $(k \times 1)$ vector of variables unique to the respondent $(t_i, r_i, Y_i,$ etc.),

β is a $(p \times 1)$ vector of parameters to be estimated, and ε_i is an i.i.d. error term. We can obtain an estimate of β via nonlinear least squares. The nonlinear least squares estimator[8] $\hat{\beta}$ is consistent so long as the error terms are independent and identically distributed, with mean zero and finite variance σ^2—assumptions that will be tested below.

Given the structure of equation (7) and the nature of the data, it is possible to estimate only two of the three parameters. With no loss of generality, set the coefficient

(10) $\quad \beta_2 = U'(Y) = 1.$

The two tests that will be possible with the model are whether utility is greater in the healthy state, or

(11) $\quad \beta_1 = U(Y) - V(Y) > 0,$

and whether ill health lowers the marginal utility of income, or

(12) $\quad \beta_3 = V'(Y) < 1.$

A test of the financial loss model of adverse health effects would be to test the joint restriction $\beta_1 > 0$ and $\beta_3 > 1$. Thus, the distinguishing test of the health state approach, as compared with the financial loss model, is inequality (12).

Table 2 presents the nonlinear least squares estimate of equation (7), where β_2 has been constrained to equal 1. The first equation in Table 2 presents the estimates for the full sample, and the next three equations report estimates for each label subsample. The utility function parameters can be viewed as averages across the sample. The final equation allows each parameter to be a linear combination of the major human capital variables, providing evidence on heterogeneity of preferences. In this case we report both the individual parameter estimates as well as the estimated β_1 and β_3 values evaluated at the sample mean.

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[8]A. Robert Gallant (1975) describes the nonlinear estimation procedure and its properties.

TABLE 2—NONLINEAR LEAST-SQUARES ESTIMATES OF FIRST-ORDER
TAYLOR'S SERIES EXPANSION

| Parameter | Coefficient Estimate, (Asymptotic Standard Error) and [Heteroscedastic Consistent Standard Error] | | | | |
	Full Sample	TNT	Asbestos	Chloroacetophenone	Full Sample
β_{10} (Intercept)					225.3
					(70.92)
					[64.25]
β_{11} (EDUC)					− 5.30
					(5.28)
					[4.72]
β_{12} (TENURE)					1.22
					(1.81)
					[1.98]
β_1	167.7	194.1	165.5	− 38.74	158.9[a]
	(11.81)	(17.07)	(15.63)	(60.81)	(17.80)
	[12.90]	[17.36]	[17.07]	[105.8]	[19.17]
β_{30} (Intercept)					0.775
					(0.826)
					[0.783]
β_{31} (EDUC)					− 0.023
					(0.043)
					[0.044]
β_{32} (TENURE)					− 0.032
					(0.018)
					[0.020]
β_3	0.773	0.818	0.415	2.522	0.859[a]
	(0.134)	(0.194)	(0.187)	(0.561)	(0.228)
	[0.163]	[0.153]	[0.173]	[1.310]	[0.225]
R^2	0.505	0.438	0.480	0.456	0.513
nR^2 Test[b]	1.469	1.123	2.166	20.19	11.20
Observations	249	78	87	84	249

[a]Evaluated at sample averages for EDUC and TENURE.
[b]This statistic for the first 4 columns is asymptotically distributed χ^2 with 3 degrees of freedom. The critical value for χ^2 (3 d.f.) at the 95 percent confidence level is 7.81. The statistic in the fifth column is asymptotically distributed as χ^2 (18 d.f.) with a 95 percent confidence level of 28.87.

The estimates of β_1 and β_3 are extremely precise and in the expected direction for all but the chloroacetophenone results. The coefficient β_1, which represents the difference between the utility when healthy and when injured, has the expected positive sign, with a coefficient that is over 10 times larger than its standard error for the first three sets of results. The β_1 coefficients for these first three columns also are not significantly different from each other. Since von Neumann-Morgenstern utility functions are defined only up to a positive linear transformation, it is only the sign of β_1 rather than its

magnitude that is of consequence. Individuals prefer the good health state, as predicted.

The coefficient of β_3 is also positive and passes tests of statistical significance at very demanding levels since the asymptotic t-ratio is almost 6. The point estimate of β_3 for the full sample is 0.773, which implies that the marginal utility of income in the ill health state is about three-fourths that of the good health state. The confidence intervals of β_3 for TNT and asbestos overlap the confidence intervals of the full sample estimate of β_3.

The most pertinent statistical test is not whether β_3 is significantly different from zero

but whether β_3 is significantly below 1, the marginal utility of income when healthy. One can reject the hypothesis that β_3 equals 1 at the 5 percent confidence level. The marginal utility of income is significantly lower in ill health than when in good health for the first three columns of estimates.

This result provides the distinguishing test between the financial loss and health state models of injuries. For all results except chloroacetophenone, there is evidence that the injury lowers welfare, so that β_1 will be positive. If this lowering takes the form of being tantamount to being a drop in income, then β_3 will exceed 1. With the health state model, the injury alters the shape of the utility function, with the most common assumption being that an injury reduces the marginal utility of income. The estimate of β_3 is in line with the health state model, and it suggests that treatment of health effects as being equivalent to monetary losses is inappropriate.

The one divergent set of results is for chloroacetophenone. The sign of β_1 is negative, which is the opposite of the expected relationship in inequality (11), but this coefficient is not statistically significant at the 5 percent level. The raised marginal utility of income after an accident ($\beta_3 = 2.522$) is consistent with the financial loss equivalent of the model rather than the health state approach. This result is not implausible since this chemical imposes no permanent health impairment. It is an eye irritant that causes tearing but does not inhibit one's ability to derive utility from additional expenditures. In Section III we will impose more structure on the estimation, which reduces but does not completely eliminate the differing performance of the chloroacetophenone group.

To explore possible heterogeneity in the parameter estimates, in the final equation in Table 2 the parameters vary across individuals according to the linear equation,

$$\beta_i = \beta_{i0} + \beta_{i1}EDUC + \beta_{i2}TENURE,$$

$$\text{for } i = 1 \text{ and } 3,$$

where worker education (EDUC) and job experience (TENURE) are good measures of

lifetime wealth.[9] Thus, for both β_1 and β_3 this equation includes estimates of this parameter and its interaction with education and job tenure. If better educated and experienced workers suffer a greater (lower) drop in utility after an injury, then the sign of the interaction with β_1 will be positive (negative). Similarly, a larger (smaller) drop in marginal utility after an accident will lead β_3 to have a negative (positive) sign.

Evaluating the sum of the parameter estimates at the sample averages, we find $\beta_1 = 158.9$ and $\beta_3 = 0.859$. These results are within one standard error of the full sample estimates in column 1 of Table 2, even when the adjusted standard errors are used.[10] Because of the large variances for the variables in the final equation, the estimated β_3 is less than one standard deviation away from the critical value of 1 that is pertinent for the testing of the hypothesis given in inequality (12) above. None of the personal characteristic variables is statistically significant at the 5 percent level (two-tailed test). The most precisely estimated interaction is the β_3 interaction with tenure (significant at the 5 percent level, one-tailed test), which suggests that more experienced workers will suffer a greater drop in marginal utility after an accident. This result is expected since the greater family responsibilities and more limited mobility of more senior workers creates a demand for greater insurance coverage after an accident, which is the substantive implication of a lower value of β_3.

One extreme hypothesis that can be tested using the results in Table 2 is whether injuries have no effect whatsoever on either the level of utility or the marginal utility of income. This hypothesis can be rejected at even very demanding confidence levels in every case shown in Table 2.

Two statistical issues must be addressed before turning to alternative specifications of

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[9] More detailed sets of interaction created convergence problems and could not be estimated.

[10] The adjusted standard errors are developed using the procedure in White and Domowitz (1984). This procedure is designed to adjust for the influence of heteroscedasticity.

the utility function. First, the i.i.d. assumption of the model may not be satisfied for this cross-sectional data base, as the error term may be heteroscedastic. Table 2 includes both the conventional standard error and the heteroscedastically consistent standard error for nonlinear models.[11] The second set of standard errors has very similar indications for statistical significance so that any heteroscedasticity that is present does not appear to be consequential.

Second, it is possible to test the i.i.d. assumption explicitly and to test the correctness of the model specification. Based on the Halbert L. White and Ian Domowitz (1984) test summarized in the Appendix, one cannot reject at the 95 percent confidence level the assumption that the errors are homoscedastic. Furthermore, one cannot reject the assumption that the first-order Taylor's series model for the full sample is a correct specification up to an independent additive error term.

B. Second-Order Taylor's Series

To test the robustness of the previous model and to explore the potential role of second-order terms, we also estimated a model based on a second-order Taylor's series. The second-order expansion will allow us to estimate $U''[\cdot]$ and $V''[\cdot]$, which can be used to calculate measures of risk aversion. The second-order expansion is substantially more difficult to solve algebraically because the expansion of $U(Y(1+\delta)(1-t_2))$ and $V(Y(1+\delta)r_2)$ about Y generate a quadratic expression in δ, our variable of interest. The second-order model can be constructed as follows. Denote the quadratic expression in δ as

$$A\delta^2 + B\delta + C = 0,$$

and define β_1, β_2 and β_3 as before. Let $\beta_{22} = U''(Y)$ and $\beta_{33} = V''(Y)$.

Some straightforward (but lengthy) algebra generates the following values for the

quadratic coefficients:

$$EU_1 = \beta_1 + (1-p_1)\left\{-\beta_2 Yt_1 + 0.5(Yt_1)^2\beta_{22}\right\}$$
$$+ p\left\{Y(r_1-1)\beta_3 + 0.5[Y(r_1-1)]^2\beta_{33}\right\},$$

$$A = 0.5(1-p_2)\left\{[Y(1-t_2)]^2\beta_{22}\right.$$
$$\left. + p_2(Yr_2)^2\beta_{33},\right.$$

$$B = (1-p_2)Y(1-t_2)(\beta_2 - Yt_2\beta_{22})$$
$$+ p_2 Yr_2\left\{\beta_3 + Y(r_2-1)\beta_{33}\right\},$$

and

$$C = \beta_1 + (1-p_2)\left\{-Yt_2\beta_2 + 0.5(Yt_2)^2\beta_{22}\right\}$$
$$+ p_2\left\{Y(r_2-)\beta_3 + 0.5(Y(r_2-1))^2\beta_{33}\right\}$$
$$- EU_1.$$

The solution to the quadratic suggests two possible roots. In preliminary analysis with this expansion, numeric calculations indicate that only one of the roots can predict positive values for δ, and therefore, the implicit equation we choose to estimate is of the form:

$$\delta = \left[-B + (B^2 - 4AC)^{1/2}\right]\Big/2A + \varepsilon.$$

As is the case with the first-order series, both β_2 and β_3 (the marginal utility terms) are not identified and so without loss of generality, we set $\beta_2 = 1$ and test whether $\beta_3 < 1$. Consumer theory suggests that both second-order terms should be negative, but there is no theoretical basis for predicting which term should be larger in absolute value.

In the first column of Table 3, we present Taylor's series results for a model where both of the second-order terms are allowed to vary. The value of β_3 drops substantially from the 0.77 estimated in the first-order case. However, we accept the hypothesis that the coefficient is significantly below the critical value of 1 at about the same confidence level as in Table 3, which is the main hypothesis of interest. The magnitude and the

TABLE 3—NONLINEAR LEAST SQUARES ESTIMATES OF SECOND-ORDER TAYLOR'S
SERIES EXPANSION

| Parameter | Coefficient Estimate, (Asymptotic Standard Error) and [Heteroscedastic Consistent Standard Error] | |
	Full Sample	Full Sample $\beta_{22} = \beta_{33}$
β_1	184.2	170.3
	(13.47)	(11.79)
	[15.63]	[12.33]
β_3	0.261	0.701
	(0.316)	(0.188)
	[0.350]	[0.189]
β_{22}	2.1E−3	−7.4E−4
	(2.4E−3)	(1.5E−3)
	[2.4E−3]	[1.1E−3]
β_{33}	−1.8E−3	−7.4E−4
	(1.6E−3)	(1.5E−3)
	[1.7E−3]	[1.5E−3]
R^2	0.509	0.506
nR^2 Test	33.86[a]	13.89[b]

[a] This statistic is asymptotically distributed χ^2 with 10 degrees of freedom. The critical value for χ^2 (10 d.f.) at the 95 percent confidence level is 18.31.
[b] This statistic is asymptotically distributed χ^2 with 6 degrees of freedom. The critical value for χ^2 (6 d.f.) at the 95 percent confidence level is 12.59.

significance of β_1 is quite similar to the first-order result. The coefficients for the second-order terms are not estimated with a great deal of precision, and we cannot reject the joint hypothesis that both terms equal zero.

Given the lack of precision in the second-order terms, we estimated a second model reported in the final column of Table 3, where the second-order terms are restricted to be equal. By restricting $\beta_{22} = \beta_{33}$, the estimates for the second-order terms are both negative but insignificant, and the estimates for β_1 and β_3 are quite similar to the first-order series results. Although the estimate for β_3 is quite different in both columns in Table 3, we cannot reject the hypothesis that the results in both columns are equal.[12] This is not surprising given the closeness of the estimates for β_1 in both columns, the impre-

cision in both second-order terms, and the large variance for β_3 in column 1.[13]

Although the second-order results are consistent with the theoretical predictions, the failure of any second-order terms to be statistically significant suggests that the earlier first-order results represent a reasonable approximation. The specification test results reinforce this conclusion.

III. Estimates with Logarithmic Utility Functions

To obtain estimates of the entire utility function shape, as opposed to simply the

[12] The test statistic is asymptotically an F with 1 and 245 degrees of freedom. The test statistic was 1.81, which is below the 95 percent critical value of 3.84.

[13] Notice also that the nR^2 test indicates possible model misspecification. The large variances for the second-order terms and the rejection of the nR^2 test are not surprising given that the survey asked for only one response. The survey was originally designed to elicit the response ΔY for a given Δp. Without placing more structure on the utility function, it may be difficult to determine second moments of utility since respondents were not asked additional questions to indicate changes in the rate of tradeoff between p and Y.

utility differences and the relative marginal utility of the two states, one must impose additional structure on the utility functions. The particular functional form we have selected is the Cobb-Douglas parameterization, where $U(Y)$ is of the form Y^u. Although this functional form does not have the same flexibility as the approximation involving the Taylor's series expansion, it is not extremely restrictive.[14] Upon taking logarithms of the within-state utility, we obtain a logarithmic utility function where $U(Y)$ equals $u[\log(Y)]$.

In the case of this model, the logarithmic formulation implies that

$$(13) \qquad U(Y) = u[\log(Y)],$$

and

$$(14) \qquad V(Y) = v[\log(Y)],$$

where u is a multiplicative parameter for the healthy state utility function and v is the parameter for the unhealthy state. If $u > v$, then the utility and the marginal utility of income are greater when the worker is in the good health state, which is the standard assumption in the literature.

The logarithmic utility function is frequently used in finance contexts[15] and in empirical applications analyzing von Neumann-Morgenstern utility functions. This function embodies decreasing risk aversion, which is a common empirical phenomenon.[16]

The major drawback of the logarithmic approach is that the utility and marginal utility will each be governed by a single parameter. This formulation in effect links the tests of whether there is a utility drop in the ill health state (i.e., $v < u$) and whether there is a marginal utility decline (i.e., $v/u < 1$), so that it does not provide an uncon-

strained test of the model. However, the overall test of behavior was the subject of the Taylor's series test, and one can view the logarithmic utility function as imposing more specific functional structure on the relationships that were shown to hold in Section II. The purpose of the additional structure is to obtain estimates that will be used in greater detail to examine attitudes toward risk. Although other functional forms for utility functions have appeared in the literature, these could not be used because of both the nature of the data and the iterative search procedure that was used.[17]

There are several types of checks on the realism of the model. Many of these checks are based on comparison with the Taylor's series estimates. First, does utility drop in the ill health state? The magnitude of any such drop is irrelevant since von Neumann-Morgenstern utility functions are unaffected by a positive linear transformation. Second, does the point estimate of the ratio of the marginal utility in ill health relative to good

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[14]Melvyn A. Fuss, Daniel McFadden, and Yair Mundlak (1978) discuss its use in production contexts.

[15]Examples of the use of logarithmic utility functions in the finance literature abound. See Albert L. Kraus and Robert H. Litzenberger (1975), Mark E. Rubinstein (1977), and Paul A. Samuelson (1969).

[16]See Keeney and Raiffa (1976).

[17]We also attempted to estimate equation (1) with two other specifications for utility: the constant risk aversion (CRA) utility function and the constant relative risk aversion (CRRA) functions. However, these models are not identified given the formulation of the problem, as presented in equation (1). The CRA utility function is typically denoted as $U = -\exp[-rY]$, where $r < 0$. given the CRA specification, we are unable to obtain a closed-form solution for δ. Instead, we attempted to estimate the implicit equation $EU_1 - EU_2 = \varepsilon$, where ε is an i.i.d. error with mean zero. Since there is no appropriate normalization of the parameters, we must estimate two variables, r_1 in a healthy state, and r_2 in an unhealthy state. Given the properties of the CRA function, the sum of squared errors is minimized where $r_1 = r_2 = 0$, which forces utility to equal one in all periods. Subsequently, the parameter values generate the equality

$$EU_1 - EU_2 = [(1-p)+p] - [(1-q)+q] = 0.$$

The CRRA utility function is defined to be $U = Y^{(1-\rho)}/(1-\rho)$, where $\rho \geq 1$. As in the previous example, we are unable to obtain a closed form solution for δ so we must estimate the implicit equation $EU_1 - EU_2 = \varepsilon$. Algorithmically, the sum of squared errors can be minimized by choosing extremely large values for both ρ_1 and ρ_2. This has the property of forcing utility in all periods to machine zero and therefore the difference, $EU_1 - EU_2$, is also zero.

health differ statistically in the Taylor's series and logarithmic utility function cases? Third, do the models provide similar estimates of $[U(Y) - V(Y)]/V'(Y)$, which is a utility difference statistic that is unaffected by positive linear transformation of the utility function? Finally, we will provide a White-Domowitz (1984) test of whether the model is correct up to an additive independent error, thus providing a formal specification test.

The requirement given by equation (5) above can be written as

$$(15) \quad (1 - p_1) u\{\log[Y(1 - t_1)]\}$$

$$+ p_1 v\{\log(Yr_1)\}$$

$$= (1 - p_2) u\{\log[Y(1 + \delta)(1 - t_2)]\}$$

$$+ p_2 v\{\log[Y(1 + \delta)r_2]\},$$

which equates the expected utility of the initial job and the transformed job.

Even in conjunction with the imposed functional form, it will not be possible to estimate both parameters, u and v. As a result, we will estimate their ratio α given by

$$(16) \quad \alpha = u/v.$$

As in the Taylor's series case, the dependent variable is δ, the percentage wage increase that the respondent requires to face an increased risk. Using the normalization of equation (16), we solve for δ to yield

$$(17) \quad \delta = \exp\left[\frac{K_1 - K_2}{(1 - p_2)\alpha + p_2}\right]$$

$$- 1 + \varepsilon,$$

where

$$K_1 = (1 - p_1)\alpha \log[Y(1 - t_1)]$$

$$+ p_1 \log[Yr_1],$$

and

$$K_2 = (1 - p_2)\alpha \log[Y(1 - t_2)]$$

$$+ p_2 \log[Yr_2].$$

We will estimate equation (17) using nonlinear least squares, again assuming the error term is i.i.d. with mean zero and finite variance.

The principal hypothesis is that

$$(18) \quad \alpha > 1,$$

or, for any given level of income, both the level of utility and the marginal utility of income are higher in the good health state.

Two cases will be considered. First, we will estimate the homogeneous preference model in which all utility functions are identical (i.e., $\alpha = \alpha_0$), thus providing an average value for α across the sample. Second, we then permit α to vary with personal characteristic variables X_i. Doing so leads to the heterogeneous preference assumption that

$$(19) \quad \alpha = \alpha_0 + \beta_1 EDUC + \beta_2 TENURE,$$

where the personal characteristic variables are education and job experience.

The estimate of the homogeneous preference model appears as equation (1) in Table 4. The estimate of α_0 is clearly statistically different from zero (asymptotic $t = 134$), but the more relevant issue is whether α_0 differs from 1.0. The estimate for the full sample that α_0 equals 1.077 lies 9.6 standard deviations above 1.0, so one can reject the hypothesis that $u = v$, indicating that both the utility level and the marginal utility are greater in the good health state.

The α_0 values for each of the chemical subsamples are also above 1.0. Both the TNT and asbestos α_0 values are not significantly different from each other or the full sample results. However, one can reject the hypothesis that all of the α_0 coefficients are identical. Nevertheless, the magnitude of the α_0 value for chloroacetophenone is not greatly different in magnitude (for example, its α_0 confidence interval overlaps with that for asbestos), and the overall structure of the utility function implied by the results is very similar to that for the other chemical subsamples. The additional structure imposed by the logarithmic utility function may have muted some of the differences across label-

TABLE 4—NONLINEAR LEAST-SQUARES ESTIMATES OF
LOGARITHMIC UTILITY MODEL

| | Coefficient Estimate, (Standard Error), and [Heterscedastic Consistent Standard Error] | | | | |
| | Model 1 | | | | Model 2 |
Parameter	Full Sample	TNT	Asbestos	Chloroaceto-phenone	Full Sample
Intercept					1.294
					(0.033)
					[0.033]
EDUC					−0.013
					(0.002)
					[0.002]
TENURE					−0.004
					(0.001)
					[0.001]
α_0	1.077	1.094	1.065	1.043	1.082[a]
	(0.008)	(0.013)	(0.012)	(0.018)	(0.016)
	[0.009]	[0.014]	[0.015]	[0.022]	[0.016]
R^2	0.363	0.147	0.208	0.436	0.466
nR^2 Test of Specification	12.00[b]	1.63[b]	6.55[b]	1.03[b]	3.13[c]
Observation	249	78	87	84	249

[a] Evaluated at sample averages for EDUC and TENURE.

[b] This test statistic is asymptotically distributed χ^2 with 1 degree of freedom. The critical value for χ^2 (1 d.f.) at the 95 percent confidence level is 3.84.

[c] This statistic is asymptotically distributed χ^2 with 6 degrees of freedom. The critical value for χ^2 (6 d.f.) at the 95 percent confidence level is 12.59.

ing groups that were apparent in the Taylor's series results.

The relative discrepancy between the marginal utility in the two health states is narrower for the logarithmic model than the first-order Taylor's series expansion. The ratio of marginal utilities, $V'(Y)/U'(Y)$, is given by $1/\alpha$ for the logarithmic model and by β_3 in the case of the Taylor's series model. The estimates are 0.93 and 0.78, respectively. In each case, the accident lowers the utility and marginal utility of income, but the Taylor's series estimates imply a greater relative gap in the marginal utilities. The 95 percent confidence intervals for $V'(Y)/U'(Y)$ for the two different estimation approaches overlap (full sample results).

Another comparison that is meaningful, given possible differences in the utility metric, is the ratio of the utility difference to the marginal utility of income when injured. One establishes a comparable metric for utility differences by dividing by a marginal utility term. The value of $[U(Y)-V(Y)]/V'(Y)$ is 216 for the full sample Taylor's series results and is 179 for the full sample logarithmic results—a difference of under 20 percent.

As in the case of the Taylor's series results, the adjusted standard errors are similar to those that have not been adjusted for heteroscedasticity. In addition, for Model 2 (but not Model 1), one cannot reject the hypothesis that the specification is correct up to an additive independent error (see Appendix). This result is not inconsistent with a similar finding for the first-order Taylor's series model since one can view the flexible form of the Taylor's series model as providing an approximation to the logarithmic formulation.

TABLE 5—SIMULATION RESULTS, SENSITIVITY OF ESTIMATES
TO OVERESTIMATE OF δ

Percent Reduction in δ in percent	Coefficient Estimates and Standard Errors				
	Taylor's Series Model			Logarithmic Model	
	β_1	β_2	R^2	α	R^2
5	158.2	0.740	0.513	1.069	0.367
	(11.25)	(0.125)		(0.007)	
10	147.9	0.707	0.522	1.061	0.377
	(10.67)	(0.116)		(0.007)	
25	118.2	0.608	0.544	1.036	0.377
	(8.89)	(0.091)		(0.006)	

The imposition of additional structure with the logarithmic model has also facilitated the estimation of variations in individual preferences. Recall from equation (19) that positive coefficients imply that the particular demographic group has a higher utility of income value and higher marginal utility of income when in good health relative to ill health. Both education and tenure have negative signs, implying that there is less of a drop in the marginal utility after an accident for these workers.

A possible bias in the model may arise if workers responded strategically to the survey, for example, by exaggerating claims of the required wage increase δ. To check our results against a possible bias in δ, we reestimate the Taylor's series and the logarithmic model after systematically depressing the response variable δ. This simple test illustrates whether our results are sensitive to exaggerated claims of δ. Table 5 reports the test results from reducing the values of δ by 5, 10, and 25 percent.

The sensitivity analysis for the logarithmic model indicates that the parameter α is sensitive to a possible bias in the response variable δ. However, even a 25 percent overestimate in δ does not alter the basic conclusion that α is significantly greater than 1. Likewise, the general character of the results for the Taylor's series case are not altered as δ is decreased. As the percentage reduction in δ is increased, the primary parameter of interest, β_3, actually declines in value, indicating that the choice between the health state and

the monetary loss model is not biased by a strategic response for δ.

IV. Economic Implications

Without knowledge of the shape of individual preferences, the domain of economic inquiry is largely limited to a single issue—the local rate of tradeoff between risk and money. Using the estimates of the logarithmic utility function, we will extend the domain of inquiry to assess how risk-money tradeoffs vary with the base level of risk, the extent of the risk change, and individual income. We will also estimate the optimal workers' compensation replacement rate. Knowledge of the utility function enables us to address a variety of concerns that have been central to the risk bearing field but which have never been addressed empirically.

A. Variation in the Implicit Value of Statistical Injury with the Base Risk

The most useful means for expressing the risk-money tradeoff is in terms of the dollar compensation required per unit of risk. This rate of tradeoff can be calculated for marginal changes of risk as it represents the value of $\partial Y/\partial p$ for a given value of expected utility.[18] At the mean risk level for the

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[18]Using the formula in Viscusi (1979, p. 12), the expected utility formulation from the left side of equation (15), and assuming that an individual works 50

TABLE 6—EFFECT OF THE BASE RISK
LEVEL ON THE IMPLICIT VALUE OF AN INJURY

Base Risk Level	Implicit Dollar Value of Injury	
	Logarithmic	Taylor's Series
0.0	13,262	11,313
0.1	13,357	11,569
0.2	13,454	11,838
0.3	13,553	12,119
0.4	13,653	12,414
0.5	13,754	12,724
0.6	13,857	13,050
0.7	13,961	13,392
0.8	14,067	13,753
0.9	14,174	14,134
1.0	14,284	14,537
0.085	13,343	11,530
(Sample Mean)		

sample of approximately 0.085, the logarith-
mic utility function estimates yield a value of
an injury of $13,343 (1982 dollars). As the
value of injury figures reported in Table 6
indicate, there is not substantial variation in
the implicit value of an injury with the base
risk level, as the range is from $13,262 to
$14,284.

The Taylor's series results can be used to
generate similar estimates of the implicit
value of injury, which is $\partial Y / \partial p$, holding
EU_1 constant.[19] The implicit value of injury

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weeks per year, the implicit value of a statistical injury
Z in the logarithmic utility function case is given by

$$Z = 50Y \frac{[\alpha \ln(Y(1-t)) - \ln Yr]}{(1-p)\alpha + p}.$$

[19] The statistic can be written as

$$Z \equiv \frac{\partial Y}{\partial p} = \frac{U(Y(1-t)) - V(Yr)}{(1-p)U'(Y(1-t)) + pV'(Yr)}.$$

The explicit characterization of Z in the Taylor's series
model is generated by using a first-order series expan-
sion to approximate $U[Y(1-t)]$ and $V[Yr]$ about Y,
and a first-order series to approximate $U'[Y(1-t)]$ and
$V'[Yr]$ about Y. Defining the parameters β_1, β_2, β_3,
β_{22}, and β_{33} as before, and assuming the worker is
employed for 50 weeks, we can write Z as

$$Z = 50 \frac{\beta_1 - Yt\beta_2 - Y(r-1)\beta_3}{(1-p)\beta_2 + p\beta_3 - (1-p)Yt\beta_{22} + pY(r-1)\beta_{33}}.$$

The calculation uses the Taylor's series results from the
second column of Table 3 and the mean risk level for p.

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for the unconstrained second-order Taylor's
series estimates is $11,530, which is some-
what below the logarithmic estimate. As the
results in the final column of Table 6 indi-
cate, the variation in the implicit value of an
injury with the risk level follows the same
general pattern as in the logarithmic case,
but is somewhat greater. Using the estimates
in which we constrain the statistically in-
significant second-order terms to equal zero
(see final column in Table 3), the implicit
value of an injury rises to $12,057. This
estimate is closer to the logarithmic utility
function result.

The variation of the injury value with the
risk level is of independent economic inter-
est. Several economic models predict that the
valuation of a risk change should be an
increasing function of the base risk level.[20]
The source of this effect is the lower oppor-
tunity cost of resources with high risk levels.
At high levels of risk, the probability of
spending the money when in good health is
less. Since the utility and marginal utility of
money is less when one is injured than when
one is healthy, for any given income level,
the additional expected utility produced by
wage compensation is reduced by increases
in the base risk.

The types of variations that are predicted
theoretically are borne out by the results in
Table 6 for both the logarithmic and Taylor's
series cases. The additional compensation
required to accept an increase in risk is
greater for high base risks. The Table 6
results also indicate a change in the tradeoff
with the base risk. These patterns follow
economic predictions, as $\partial Z / \partial p > 0$ and
$\partial^2 Z / \partial p^2 > 0$.

B. *Income Elasticity of the Value
of an Injury*

On a theoretical basis, the value of an
injury should increase with individual in-
come and wealth, and available labor market

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[20] For discussion of this and related issues, see
Viscusi (1979) and Weinstein, Shepard, and Pliskin
(1980).

data are consistent with this relationship (see Viscusi (1978, 1979)). However, the extent of the observed income effects have not been large since existing data sets are not well-suited to disentangling the role of compensating differentials for risk and income effects that govern job choice.

At the mean value of an injury for the sample, the income elasticity of the value of an injury in the logarithmic case is 1.0995.[21] Thus, the value of an injury is roughly proportional to one's base income level. Using Taylor's series results from the final column in Table 3, the parameter estimates suggest that the elasticity in the Taylor's series case is approximately 0.67.[22] In the health insurance context, estimated income elasticities are generally lower—typically 0.5 or less.[23] One might expect the health insurance income elasticity to be below the elasticity of the value of an injury since demand will be muted to the extent that additional health expenditures have a diminishing, probabilistic effect on one's well-being.

[21] For the logarithmic case, the income elasticity ε of the value of an injury is given by

$$\varepsilon = \frac{Y}{Z} \frac{\partial Z}{\partial Y} = 1 + \frac{\alpha - 1}{\alpha \ln(Y(1-t)) - \ln(Yr)},$$

which is obtained by using the value of Z from fn. 18.

[22] We can calculate ε for the second-order Taylor's series model by differentiating Z with respect to Y and multiplying by Y/Z. Assuming $U'''(\cdot) = V'''(\cdot) = 0$, the value for ε can be written as

$$\varepsilon = Y \frac{\beta_2 - \beta_3 - tY\beta_{22} - Y(r-1)\beta_{33}}{\beta_1 - tY\beta_2 - Y(r-1)\beta_3}$$

$$- \frac{Y}{(1-p)\beta_2 + p\beta_3 - (1-p)tY\beta_{22} + Y(r-1)\beta_{33}}.$$

Given the imprecision with which the second-order terms are estimated, we use the results from column 2 of Table 3 where β_{22} is restricted to equal β_{11} to calculate the value for Z. Income elasticities cannot be derived using the first-order results.

[23] The income elasticity for health insurance estimates and the underlying theory are discussed in Joseph P. Phelps (1973), Charles E. Newhouse and Phelps (1976), and Phelps (1987).

The estimates of the income elasticity of injuries indicate that the value placed on individual health is not a constant, but exhibits substantial heterogeneity (see Viscusi, 1979, 1983). Knowledge of the income elasticity of the value of statistical injuries is likely to be particularly useful in the valuation of government programs with long-term effects since the growth in income over time will boost the value of the risks reduced, offsetting much of the influence of discounting.

C. The Value of Non-Incremental Risk Changes

In some cases the risk change that must be valued does not involve a small incremental change in the probability. Although medical contexts create the greatest opportunities for quantum changes in the risk level, changes in large individual risks resulting from government regulation (for example, seatbelt use requirements) pose similar problems.

From an economic standpoint, individuals should exhibit a diminishing marginal valuation of risk reduction and an increasing marginal acceptance price for risk increases. Market risk data do not enable one to address these issues since the observed risk changes tend to be small.[24]

Knowledge of utility functions enables one to make such assessments, as Table 7 summarizes the value of non-incremental risk changes from the starting point of the mean injury risk of 0.085. The purchase of a risk reduction of −0.085 is tantamount to complete elimination of the risk. Using the logarithmic estimates, there is an associated value per unit risk reduction of $12,865 for such a complete elimination of the risk. Similarly, there is a $8,989 value for the first-order Taylor's series estimates. At the opposite

[24] The predicted pattern of behavior has been borne out in an experimental consumer context by W. Kip Viscusi, Wesley A. Magat, and Joel C. Huber (1987). In their study, the marginal valuations of successive risk reductions were elicited directly, whereas here we will estimate the value of non-incremental changes using the estimated utility function.

TABLE 7—DEPENDENCE OF THE IMPLICIT VALUE OF AN INJURY ON THE EXTENT
OF THE RISK CHANGE

Risk Increment from a Sample Mean (0.085)	Implicit Dollar Value of an Injury	
	Logarithmic	Taylor's Series
−0.085	12,865	8,989
−0.050	13,059	9,131
−0.010	13,286	9,299
+0.010	13,401	9,386
+0.050	13,637	9,563
+0.250	14,918	10,566
+0.500	16,792	12,157
+0.750	19,041	14,314
+0.915	20,777	16,213

extreme, workers would require a risk-dollar tradeoff of $20,777 (logarithmic) or $16,213 (Taylor's series) to incur an increase in the injury probability from 0.085 to 1.0, or a risk increase of +0.915.

These results suggest how non-incremental risk changes differ in value from estimates based on small marginal changes in the risk. The value of an injury of $13,343 (logarithmic) for incremental changes at the sample mean is only 4 percent larger than the injury value associated with complete elimination of the risk since the initial injury probability is close to zero. The injury value associated with risk increases to a risk of 1.0 is 56 percent greater (logarithmic) than the value at the mean since the risk change is quite substantial. In addition, the change in the value of an injury increases at an increasing rate as the risk level rises. For example, the implicit value of an injury (logarithmic) rises at 1.67 times the rate over the interval (+0.750, +0.915), as compared with the interval that it did over (0.0, +0.250). A similar pattern is observed in the Taylor's series case. Individuals demand increasingly large prices per unit risk for successive risk increases and are willing to pay successively smaller amounts for additional risk decreases, as predicted.[25]

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[25] See Viscusi, Magat, and Huber (1987) for a derivation and review of the antecedents in the literature. Milton C. Weinstein, Donald S. Shepard, and Joseph Pliskin (1980) present a related discussion for fatality risks.

D. The Optimal Workers' Compensation Earnings Replacement Rate

At present, the workers' compensation earnings replacement rate is based on an algorithm that typically set the benefit equal to two-thirds of the worker's gross wage rate, subject to certain minimum benefit levels, maximum benefit levels, and benefit duration amounts. Since the marginal utility of income is reduced by an injury, as our results for both the Taylor's expansion and logarithmic model indicate, less than full earnings replacement is desirable.

How much earnings replacement is desirable cannot be determined based on available labor market data.[26] Using worker utility functions, a precise assessment is possible. In particular, suppose that workers must purchase workers' compensation through an insurance market. If the risk of injury is p, then the price of actuarially fair insurance is $p/(1-p)$ and the cost of buy-

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[26] The most that can be done is to assess the wage offset that workers are willing to accept in return for workers' compensation benefits and compare this offset with what would be observed if insurance were optimal. The estimates in Viscusi and Moore (1987) imply that the levels of benefits were suboptimal in the 1970s, but the extent of the suboptimality could not be determined. Estimates for the 1980s in Moore and Viscusi (forthcoming) indicate that substantial increases in the benefit levels since the 1970s have led to a situation in which current replacement rates are close to the optimal level.

ing insurance in the healthy state that yields payoff rY when injured is $prY/(1-p)$. If there is some insurance loading factor $h(h \geq 1)$ to cover administrative costs and a return to the insurance industry, then the cost becomes $hprY/(1-p)$.

The task of ascertaining the optimal insurance policy in the logarithmic utility function case is to

$$\underset{r}{\text{Max}}\, V = (1-p)\,\alpha \ell n\left\{ Y\left[1-\left(hpr/(1-p)\right)\right]\right.$$

$$\left.(1-t)\right\} + p\ell n(Yr).$$

After substituting for the appropriate numerical values and taking the partial derivative with respect to r, one obtains the result that

$$hr = 0.85.$$

If workers' compensation were provided on an actuarially fair basis, the optimal replacement rate would be 85 percent. Less than full earnings replacement is desirable since the marginal utility of income is lower in the ill health state. Taking into account the role of taxes, an earnings replacement of 0.85 of gross earnings does replace most of the worker's after tax income.

Under the current workers' compensation system, administrative costs are nontrivial, so that after the insurance loading costs are taken into account workers receive 80¢ for each dollar contributed, or for each dollar of benefits they pay $1.25 in premiums (i.e., $h = 1.25$). After taking these costs into account, the optimal replacement rate is 0.68.

The current workers' compensation formulas that provide for two-thirds wage replacement are close to optimal, given the role of administrative costs. The role of benefit caps and other provisions, however, reduces the effective replacement rate to only 0.64, which is slightly below this amount. In addition, if our reference point for benefit provision is what would be optimal if there were actuarially fair insurance available, then there is a much more substantial divergence from the optimal amount.

V. Conclusion

Analyses of risky decisions using market-based data are by necessity restricted to utilizing the information generated by the observed local tradeoff revealed in the market. Although this literature has yielded many profitable insights, the domain of inquiry has been substantially limited.

In this paper we explored the implications of knowing two wage-risk combinations along the individual's indifference map. This information was developed based on a survey of worker responses to the risks indicated by hazard warnings. The overall objective was to assess individuals' utility functions for good health and ill health, which will convey much more information about the character of individual preferences than the local tradeoff.

The two approaches that were used—a Taylor's series expansion with respect to a general functional form and a logarithmic utility function—each yielded similar results. Since being injured will clearly reduce the level of utility, the main question of interest is how the marginal utility of income is affected by an injury. In each case, the marginal utility of a given level of income was greater when healthy than when injured.

This result has fundamental implications for the optimal level of insurance since it implies that less than full insurance of income losses is optimal. This type of result has played a major role in the health economics and social insurance literature, but except in the case of death, the empirical foundation for making this determination has been lacking.

Even more striking is that the estimates of the logarithmic utility function enable us to ascertain not only whether less than full insurance is optimal but also what the optimal level of insurance is. In particular, we showed that the optimal earnings replacement rate for workers' compensation is 85 percent if insurance is provided on an actuarially fair basis and 68 percent if insurance is provided at the current degree of insurance loading. In each case, current benefit levels are slightly suboptimal, as has been shown using a different methodology by W.

Kip Viscusi and Michael J. Moore (1987), but in this case we can ascertain the extent of the divergence from optimality as well.

Knowledge of the utility function shape enables us to address a variety of other issues that have long been the subject of theoretical inquiry and empirical speculation. Perhaps the most striking result is the income elasticity of the value of an injury, which was found to range from 0.67 (Taylor's series) to roughly 1.0 (logarithmic case). This result enables one to make precise distinctions with respect to the heterogeneity in the value of risk-dollar tradeoffs across income groups. The greatest policy importance of this result is with respect to deferred risks since it indicates that the injury value figures used for deferred risk reductions should take into account the income growth of those affected by the regulations, leading to an adjustment that will serve to mute much of the role of discounting. These results may prove to be particularly useful in assessing the value of risk reduction to future generations.

There were two other types of concerns for which we obtained new results because of our focus on risk-dollar tradeoffs in more than a local region. As predicted by several theoretical analyses, increases in the base risk reduced the implicit value of an injury. The empirical sensitivity of the results to the base risk level was not, however, great.

Of much greater consequence was the change in the implicit value of an injury with the magnitude of any non-incremental risk change. Implicit values of an injury associated with the purchase of risk reductions diminished at an increasing rate as the extent of the risk reduction increased, and the implicit values associated with compensation for a risk increase rose at an increasing rate as the extent of the non-incremental risk change increased. As with the earlier results, these patterns are consistent with a rational economic choice model and lie outside the scope of concerns that can be addressed using market data.

Analysis of the survey experiment on worker responses to changes in their job risk has greatly expanded the range of risk-dollar tradeoffs that can be addressed empirically.

The most reassuring aspect of the findings is that even the more refined predictions of the expected utility model with health state-dependent utility functions are borne out. Knowledge of the utility function shape also enables us to address for the first time many issues that have played a central role with respect of the economic performance and optimal government policies in contexts involving risk.

APPENDIX
SPECIFICATION TESTS

In this appendix we will summarize the results of the White-Domowitz (1984) specification tests. These tests have two objectives. First, they provide a formal test of the homoscedasticity assumption. Second, they provide a test of whether the model specification is correct up to an additive error term.

Consider first the logarithmic case. To perform the test, we must first define some terms. Let $\hat{\varepsilon}_i = \delta - f(x_i, \hat{\beta})$, and let $g_{ij}(\hat{\beta})$ be the jth element of the gradient $\partial f(X_i, \hat{\beta})/\partial \beta$, where the gradient is evaluated at the estimated parameter vector, $\hat{\beta}$. Define the vector ϕ_i to be formed by all nonredundant cross products of the gradient, $g_{ij}(\hat{\beta})g_{ik}(\hat{\beta})$, for $i, j \in (1, 2, \ldots, p)$. By definition, ϕ_i has a maximum length of $p(p+1)/2$. Let n equal the number of observations. The test statistic is generated from the regression of the square of the predicted residual on the vector ϕ_i and a constant,

$$\hat{\varepsilon}_i^2 = \gamma_0 + \phi_i \gamma,$$

where γ is a $p(p+1)/2 \times 1$ vector of parameters to be estimated. The test statistic is formed by multiplying the number of observations times the (constant adjusted) R^2 of the above regression. The statistic is distributed as a χ^2 with $p(p+2)/2$ degrees of freedom. If nR^2 is less than the critical value of the χ^2 distribution, one accepts the null hypothesis of no heteroscedasticity.

The nR^2 test is also a test of the model specification. Rejection of the null hypothesis can be due either to heteroscedasticity or model misspecification. If one accepts the null hypothesis, then the model is correct up to an independent additive error term. In the first-order Taylor's series case, the nR^2 statistic of 1.469 for the full sample is well below the critical value of 7.81 at the 95 percent confidence level. The TNT and asbestos subsamples also have nR^2 values below the critical level, but the chloroacetophenone sample does not. Except for the chloroacetophenone subsample results, heteroscedasticity is not a problem, and there is also no evidence of statistically significant misspecification of the model.

The results for the logarithmic case, which are summarized at the bottom of Table 4, are similar. Consider the full sample results. The nR^2 statistics are 12.00 for Model 1 and 12.82 for Model 2, which is above the critical 95 percent confidence levels of 3.84 for Model 1 but below the critical level of 23.69 for Model 2. Simi-

larly, the Model 1 results for asbestos are below the critical statistic for TNT and asbestos is not. Thus, one cannot reject either (i) the assumption of homoscedastic errors or (ii) the assumption that the model specification is correct up to an additive error term for Model 2 or for two subsample estimates of Model 1 (TNT and chloroacetophenone).

Although we cannot reject the hypothesis that both models are correctly specified, this result is not contradictory since we can consider the Taylor's series as simply approximating the logarithmic function. Additional evidence for this conclusion is in the closeness of the two estimates β_3 and $1/\alpha$.

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